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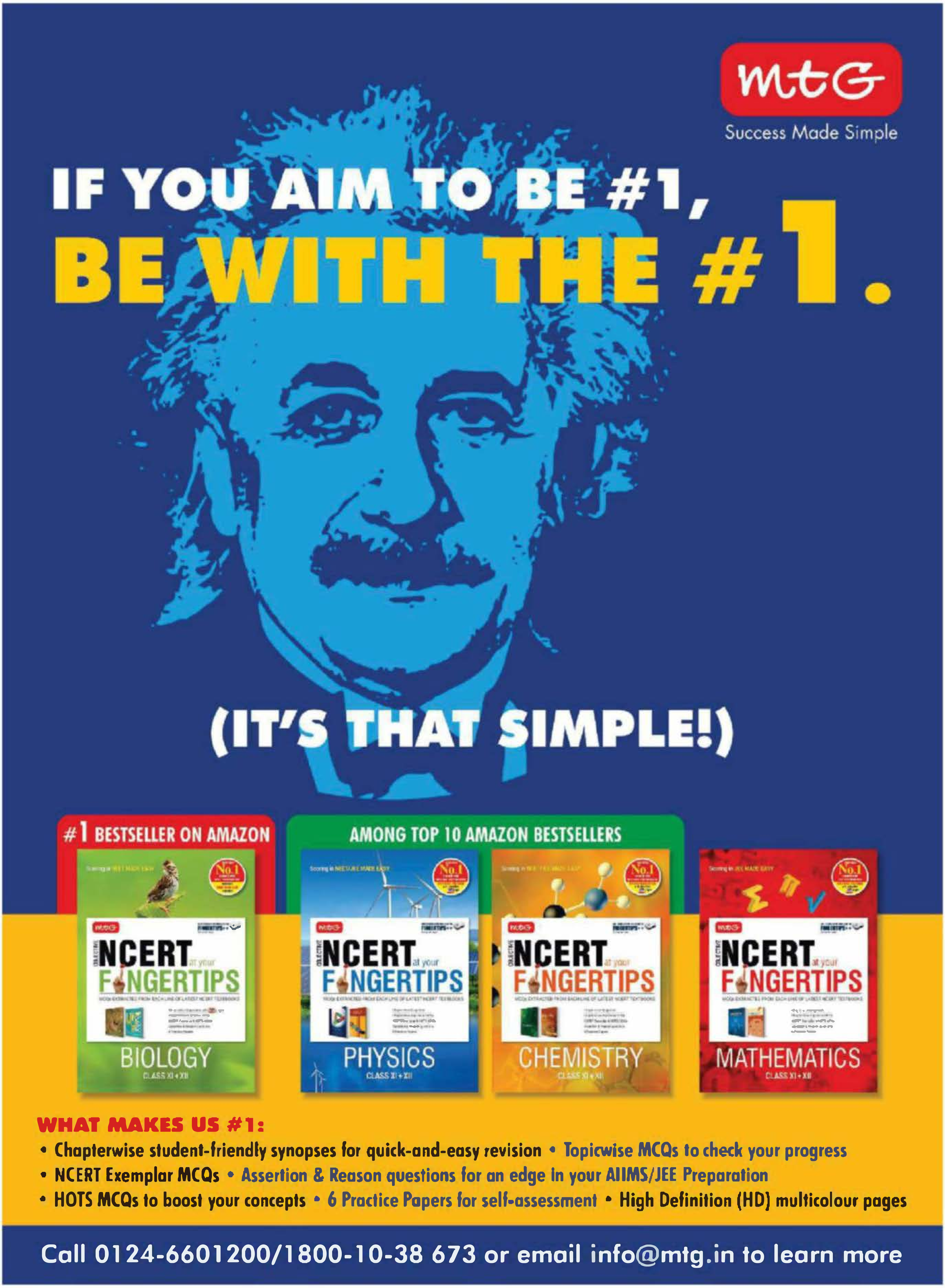

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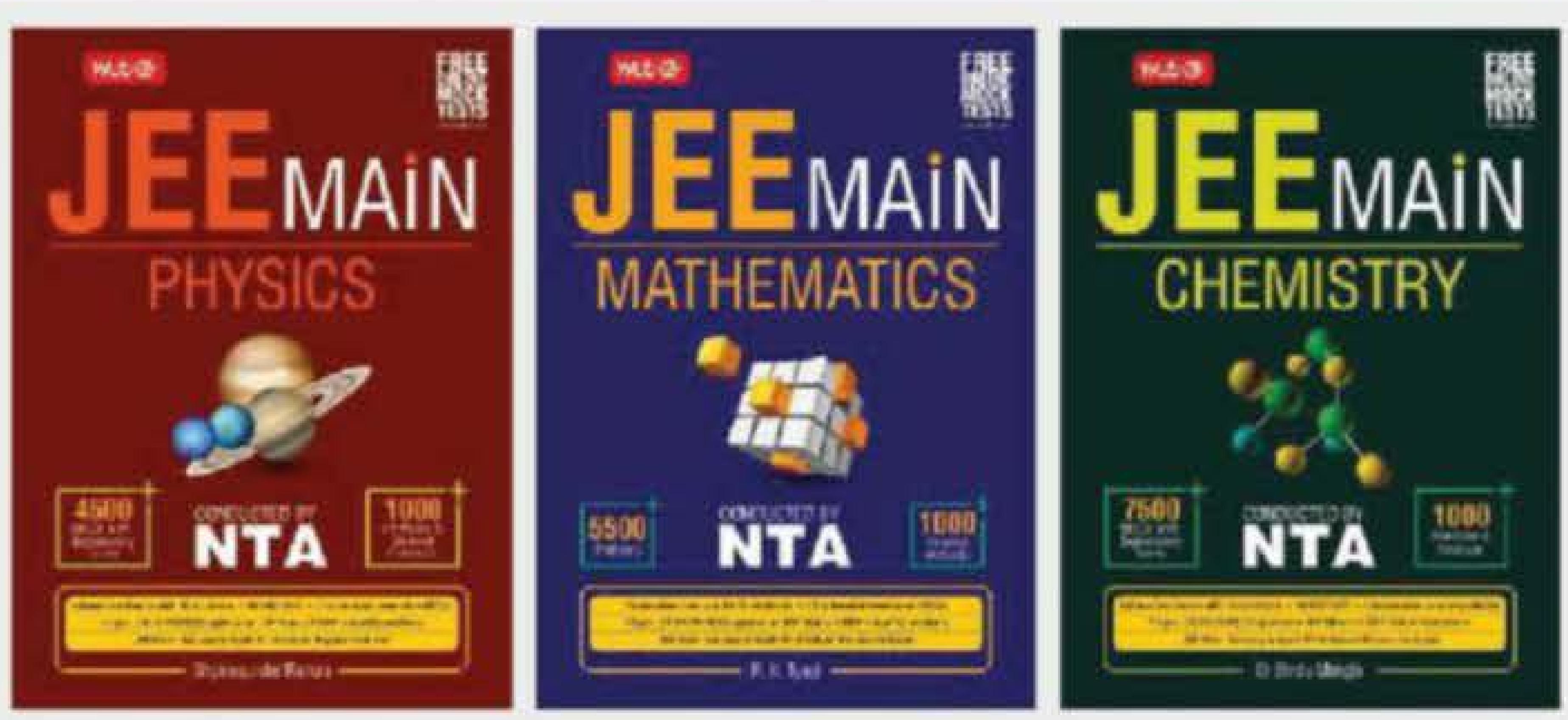
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GET SET GO *for* NEET

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1. A pan with a set of weights is attached with a light spring. When disturbed, the mass-spring system oscillates with a time period of 0.6 s. When some additional weights are added then time period is 0.7 s. The extension caused by the additional weights is approximately given by
(a) 1.38 cm (b) 3.5 cm (c) 1.75 cm (d) 2.45 cm

2. Two metal spheres are falling through a liquid of density 2×10^3 kg m^{-3} with same uniform speed. The material density of sphere 1 and sphere 2 are 8×10^3 kg m^{-3} and 11×10^3 kg m^{-3} respectively. The ratio of their radii is
(a) $\frac{11}{8}$ (b) $\sqrt{\frac{11}{8}}$ (c) $\frac{3}{2}$ (d) $\sqrt{\frac{3}{2}}$

3. A planet moves in an elliptical orbit around the sun. The eccentricity of the orbit is e. The ratio of maximum velocity to minimum velocity of the planet is
(a) $\frac{1+e}{1-e}$ (b) $\frac{1-e}{1+e}$ (c) $\frac{e}{1-e}$ (d) $\frac{e}{1+e}$

4. In the following equation, x, t and F represent displacement, time and force respectively.
$$F = a + bt + \frac{1}{c + xd} + A \sin(\omega t + \phi)$$
The dimensional formula for Ad is
(a) $[M^0 L^0 T^{-1}]$ (b) $[M^0 L^{-1} T^0]$
(c) $[M^{-1} L^0 T^0]$ (d) $[M^1 L^{-1} T^1]$

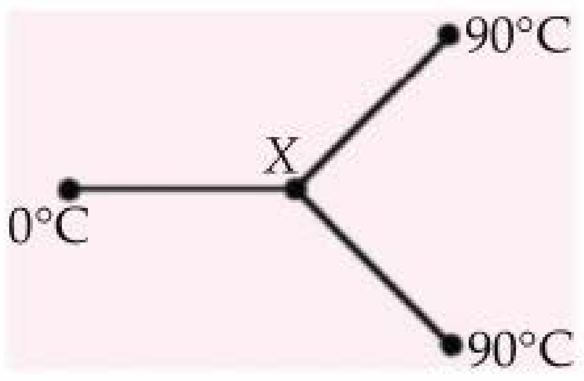
5. A stone is dropped from a height h. It hits the ground with a certain momentum p. If the same stone is dropped from a height 100% more than the previous height, the momentum when it hits the ground will change by
(a) 68% (b) 41% (c) 200% (d) 100%

6. A bullet of mass 10 g leaves a rifle at an initial velocity of 1000 m s^{-1} and strikes a target at the same level with a velocity of 500 m s^{-1}. The work done in joule to overcome the resistance of air will be
(a) 5000 (b) 3750 (c) 500 (d) 375

7. If pressure at half the depth of a lake is equal to 2/3 of the pressure at the bottom of the lake then, what is the depth of the lake?
(a) 10 m (b) 20 m (c) 60 m (d) 30 m

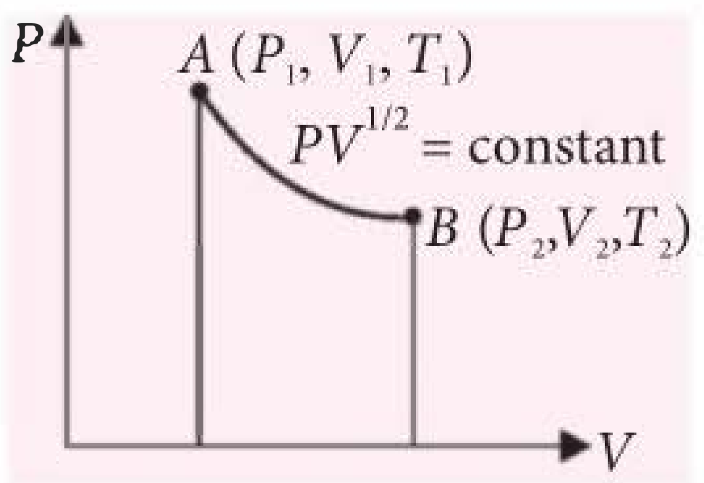
8. Three metal rods of the same material and identical in all respects are joined as shown in the figure.

The temperatures at the ends are maintained as indicated. Assuming no loss of heat from the curved surfaces of the rods, the temperature at the junction X would be



(a) 45°C (b) 60°C (c) 30°C (d) 20°C

9. 1 mole of an ideal gas in a cylindrical container have the PV diagram as shown in figure. If $V_2 = 4V_1$, then the ratio of temperatures T_1/T_2 will be



(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{2}$ (d) $\frac{3}{4}$

10. Two particles A and B are projected with same speed so that the ratio of their maximum heights reached is 3 : 1. If the speed of A is doubled without altering other parameters, the ratio of the horizontal ranges obtained by A and B is

(a) 1 : 1 (b) 2 : 1 (c) 4 : 1 (d) 3 : 2

11. A particle executes simple harmonic motion with a time period of 16 s. At time t = 2 s, the particle crosses the mean position while at t = 4 s, its velocity is 4 m s^{-1}. The amplitude of motion in metre is

(a) $\sqrt{2}\pi$ (b) $16\sqrt{2}\pi$
(c) $24\sqrt{2}\pi$ (d) $32\sqrt{2}/\pi$

12. Two sources A and B are sending notes of frequency 680 Hz. A listener moves from A to B with a constant velocity u. If the speed of sound in air is 340 m s^{-1}, what must be the value of u so that he hears 10 beats per second?

(a) 2.0 m s^{-1} (b) 2.5 m s^{-1}
(c) 3.0 m s^{-1} (d) 3.5 m s^{-1}

13. Two roads cross at right angles at O. A person A walking along one of the road at 3 m s^{-1} sees another person B walking at 4 m s^{-1} along the other road, when he is 10 m off from O. They are nearest to each other when A has walked a distance

(a) 1.8 m (b) 2 m (c) 3.0 m (d) 3.6 m

14. The bob of a simple pendulum of length 1.2 m has a velocity of 7 m s^{-1} when it is at the lowest point. The bob would leave the circular path above the centre at a height

(a) 1.0 m (b) 0.867 m
(c) 0.652 m (d) 0.512 m

15. C_P and C_V are specific heats at constant pressure and constant volume respectively. It is observed that $C_P - C_V = a$ for hydrogen gas and $C_P - C_V = b$ for nitrogen gas. So the relation between a and b is

(a) $a = 28b$ (b) $a = \frac{1}{14}b$
(c) $a = b$ (d) $a = 14b$

SOLUTIONS

1. (b): $2\pi\sqrt{\frac{m}{k}} = 0.6$ ...(i)

$2\pi\sqrt{\frac{m+m'}{k}} = 0.7$ ...(ii)

Dividing (ii) by (i), we get $\left(\frac{7}{6}\right)^2 = \frac{m+m'}{m}$

$\Rightarrow \frac{m+m'}{m} = \frac{49}{36}$

or $\frac{m+m'}{m} - 1 = \frac{49}{36} - 1 \Rightarrow \frac{m'}{m} = \frac{13}{36} \Rightarrow m' = \frac{13m}{36}$

Also, from (i), we get $\frac{k}{m} = \frac{4\pi^2}{(0.6)^2}$

Desired extension $= \frac{m'g}{k} = \frac{13}{36} \times \frac{mg}{k}$

$= \frac{13}{36} \times 10 \times \frac{0.36}{4\pi^2} \simeq 3.5$ cm.

2. (d): The terminal velocity of the spherical body of radius R and density ρ falling through a liquid of density σ is given by

$$v_T = \frac{2}{9}\frac{R^2(\rho-\sigma)g}{\eta}$$

where η is the coefficient of viscosity of the liquid.

$\therefore v_{T_1} = \frac{2R_1^2(\rho_1-\sigma)g}{9\eta}$ and $v_{T_2} = \frac{2R_2^2(\rho_2-\sigma)g}{9\eta}$

According to the given problem, $v_{T_1} = v_{T_2}$

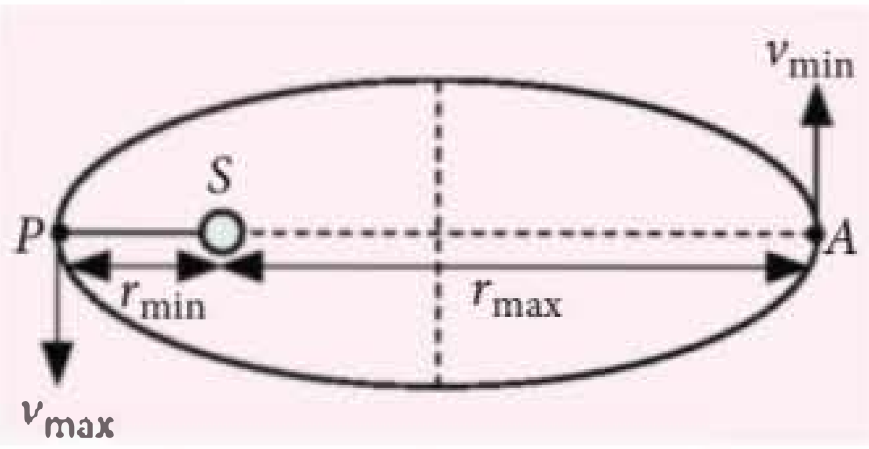
$\therefore R_1^2(\rho_1-\sigma) = R_2^2(\rho_2-\sigma)$ or $\frac{R_1^2}{R_2^2} = \frac{\rho_2-\sigma}{\rho_1-\sigma}$

Substituting the given values, we get

$$\frac{R_1^2}{R_2^2} = \frac{(11\times10^3 - 2\times10^3)}{(8\times10^3 - 2\times10^3)} = \frac{9}{6} = \frac{3}{2}$$

$$\frac{R_1}{R_2} = \sqrt{\frac{3}{2}}$$

3. (a): Let m be the mass of the planet revolving around the sun.



According to the law of conservation of angular momentum

$m v_{max} r_{min} = m v_{min} r_{max}$

$\therefore \quad \frac{v_{max}}{v_{min}} = \frac{r_{max}}{r_{min}}$ ...(i)

For an ellipse, $e = \frac{r_{max} - r_{min}}{r_{max} + r_{min}}$

$\therefore \quad 1 + e = 1 + \frac{r_{max} - r_{min}}{r_{max} + r_{min}} = \frac{2r_{max}}{r_{max} + r_{min}}$

and $1 - e = 1 - \frac{r_{max} - r_{min}}{r_{max} + r_{min}} = \frac{2r_{min}}{r_{max} + r_{min}}$

$\therefore \quad \frac{1-e}{1+e} = \frac{r_{min}}{r_{max}}$ ...(ii)

From eqns. (i) and (ii), we get $\frac{v_{max}}{v_{min}} = \frac{1+e}{1-e}$

4. (b): Given : $F = a + bt + \frac{1}{c + xd} + A \sin(\omega t + \phi)$

In a correct dimensional equation, every term has the same dimensions.

In the term $A \sin(\omega t + \phi)$, $\sin(\omega t + \phi)$ has no dimension.

$\therefore$ The dimensions of A are that of F.

$\therefore \quad [A] = [F] = [M^1 L^1 T^{-2}]$

Similarly in $\frac{1}{c + xd}$, c and xd have the same dimensions as that of $\frac{1}{F}$.

$\therefore \quad \frac{1}{c} = F$

$\therefore \quad [c] = [xd] = \left[\frac{1}{F}\right] = \frac{1}{[M^1 L^1 T^{-2}]} = [M^{-1} L^{-1} T^2]$

$\therefore \quad [d] = \left[\frac{c}{x}\right] = \frac{[M^{-1} L^{-1} T^2]}{[L^1]} = [M^{-1} L^{-2} T^2]$

$\therefore$ The dimensional formula for Ad is

$[Ad] = [M^1 L^1 T^{-2}] \times [M^{-1} L^{-2} T^2] = [M^0 L^{-1} T^0]$

5. (b): When a stone is dropped from a height h, it hits the ground with velocity $v_1 = \sqrt{2gh}$

$\therefore$ Momentum $p_1 = mv_1 = m\sqrt{2gh}$ ...(i)

When the same stone is dropped from a height $2h$ (100 % more than the previous height), it hits the ground with velocity $v_2 = \sqrt{2g \cdot 2h}$

$\therefore$ Momentum $p_2 = mv_2 = m\sqrt{2 \cdot (2gh)} = \sqrt{2} p_1$ (Using (i))

Percentage change in momentum

$= \left(\frac{p_2 - p_1}{p_1}\right) \times 100\% = \left(\frac{\sqrt{2} p_1 - p_1}{p_1}\right) \times 100\%$

$= (\sqrt{2} - 1) \times 100\% = (1.41 - 1) \times 100\% = 41\%$

6. (b): Work done by a force

= Change in kinetic energy of the body

$dW = KE_2 - KE_1$

$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = \frac{1}{2} m (v_2^2 - v_1^2)$

In this case, $m = 10$ g $= 10 \times 10^{-3}$ kg $= 10^{-2}$ kg and $v_1 = 1000$ m s^{-1} and $v_2 = 500$ m s^{-1}

$\therefore \quad dW = \frac{1}{2} \times 10^{-2} [(500)^2 - (1000)^2]$

$= \frac{1}{2} \times 10^{-2} [25 \times 10^4 - 100 \times 10^4]$

$= -\frac{1}{2} \times 10^{-2} \times 75 \times 10^4 = -3750$ J

The work done by the bullet is negative and hence the work done on the bullet by the air resistance is positive. Therefore, $W = 3750$ J.

7. (b): Pressure at half the depth $= P_0 + \frac{h}{2}\rho g$

Pressure at the bottom $= P_0 + h\rho g$

According to the given condition

$P_0 + \frac{h}{2}\rho g = \frac{2}{3}(P_0 + h\rho g)$

$\Rightarrow \quad 3P_0 + \frac{3h}{2}\rho g = 2P_0 + 2h\rho g$

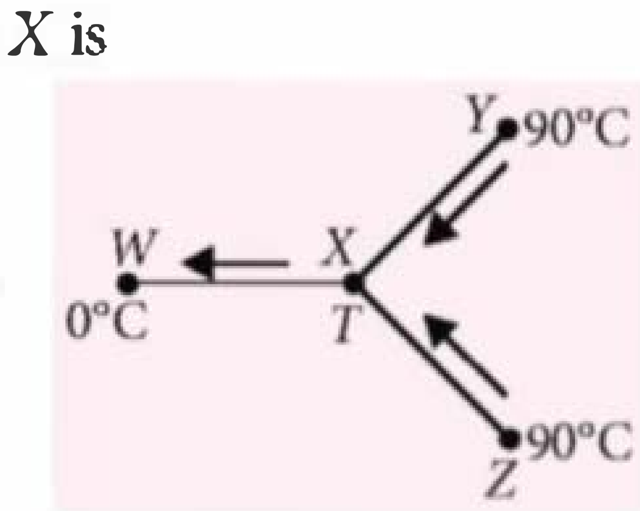
$\Rightarrow \quad h = \frac{2P_0}{\rho g} = \frac{2 \times 1.01 \times 10^5}{10^3 \times 10} = 20$ m.

8. (b): Let T be the temperature at the junction.

Let L and A be the length and area of cross-section of each rod respectively.

$\therefore$ Heat current from Y to X is

$H_1 = \frac{KA(90°C - T)}{L}$



Heat current from Z to X is

$H_2 = \frac{KA(90°C - T)}{L}$

Heat current from X to W is

$H_3 = \frac{KA(T - 0°C)}{L}$

At the junction X,

$H_1 + H_2 = H_3 \therefore\ 90°C - T + 90°C - T = T$

or $3T = 180°C$ or $T = 60°C$

9. (a): Ideal gas equation, $PV = nRT$

or $T = \dfrac{PV}{nR}$ ... (i)

According to the question, $PV^{1/2}$ = constant (A)

Multiplying both side by $\sqrt{V}$

or $PV = A\sqrt{V}$ ... (ii)

From eqns. (i) and (ii)

$$T = \frac{A\sqrt{V}}{nR} \Rightarrow T \propto \sqrt{V}$$

Now $\dfrac{T_1}{T_2} = \sqrt{\dfrac{V_1}{V_2}}$ $[\because\ V_2 = 4V_1]$

$\Rightarrow \dfrac{T_1}{T_2} = \dfrac{1}{2}$

10. (c): Let θ_1 and θ_2 be the angles of projection of the two particles A and B and u be their velocity of projection. As per question,

$$\frac{H_1}{H_2} = \frac{u^2 \sin^2\theta_1/2g}{u^2 \sin^2\theta_2/2g} = \frac{\sin^2\theta_1}{\sin^2\theta_2} = \frac{3}{1}$$

or $\dfrac{\sin\theta_1}{\sin\theta_2} = \sqrt{3} = \tan 60° = \dfrac{\sin 60°}{\sin 30°}$

$\therefore\ \theta_1 = 60°$ and $\theta_2 = 30°$

When speed of A is doubled *i.e.*, $2u$, then

$$\frac{R_1}{R_2} = \frac{(2u)^2 \sin 2\theta_1/g}{u^2 \sin 2\theta_2/g} = \frac{4\sin(2\times 60°)}{\sin(2\times 30°)} = 4$$

$\therefore\ R_1 : R_2 = 4 : 1$

11. (d): At $t = 2$ s, the particle crosses mean position.

At $t = 4$ s, its velocity is 4 m s^{-1}

For simple harmonic motion, $y = a \sin \omega t$

$\therefore\ y = a\sin\left(\dfrac{2\pi}{T}\right)t$

$y_1 = a\sin\left[\left(\dfrac{2\pi}{16}\right)\times 2\right] = a\sin\left(\dfrac{\pi}{4}\right) = \dfrac{a}{\sqrt{2}}$ ...(i)

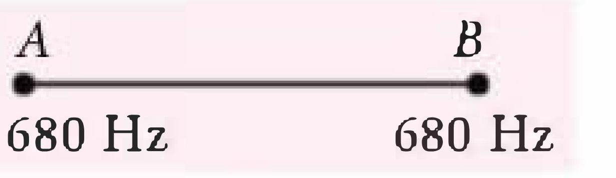
At 4 s or after 2 s from mean position,

$y_1 = \dfrac{a}{\sqrt{2}}$, velocity = 4 m s^{-1}

$\therefore$ Velocity $= \omega\sqrt{a^2 - y_1^2}$

$\Rightarrow 4 = \left(\dfrac{2\pi}{16}\right)\sqrt{a^2 - \dfrac{a^2}{2}} = \dfrac{32\sqrt{2}}{\pi}$ m. (Using (i))

12. (b): Listener moves from A to B with velocity u. The apparent frequency of sound heard by the listener from source A



$$\upsilon' = \upsilon\left(\frac{v - v_0}{v + v_s}\right) = 680\left(\frac{340 - u}{340 + 0}\right)$$

The apparent frequency of sound heard by the listener from source B

$$\upsilon'' = \upsilon\left(\frac{v + v_0}{v - v_s}\right) = 680\left(\frac{340 + u}{340 - 0}\right)$$

But listener hears 10 beats per second.

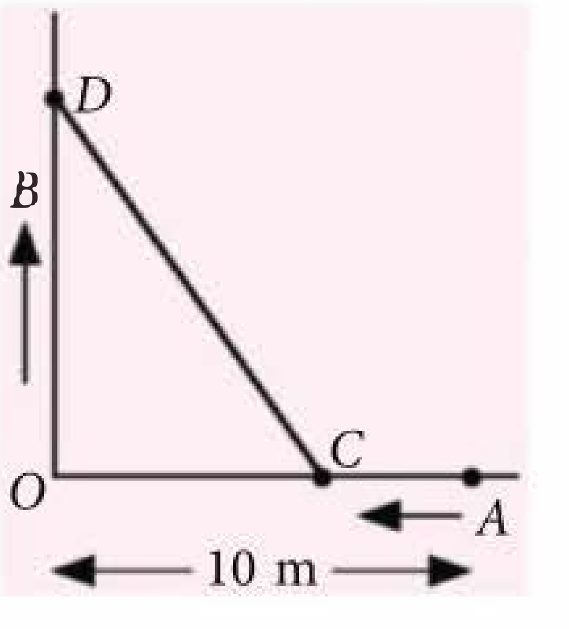
Hence, $\upsilon'' - \upsilon' = 10$

$\Rightarrow 680\left(\dfrac{340 + u}{340}\right) - 680\left(\dfrac{340 - u}{340}\right) = 10$

$= 2(340 + u - 340 + u) = 10 \Rightarrow u = 2.5$ m s^{-1}

13. (d): Let after time t_1 second when A is at C, B reaches at D. Then the distance CD is least.

Therefore $AC = 3\,t$ metres, $OD = 4\,t$ metres and $OC = (10 - 3\,t)$ metres



$\therefore\ CD^2 = OC^2 + OD^2$

$= (10 - 3\,t)^2 + (4\,t)^2$

$= 25\,t^2 - 60\,t + 100$

or $CD = [(5\,t - 6)^2 + 64]^{1/2}$

CD is least if $(5\,t - 6)^2 = 0$

or $5\,t - 6 = 0$

or $5\,t = 6$ or $t = 6/5 = 1.2$ s

So, $AC = 3 \times 1.2 = 3.6$ m

14. (b)

15. (d): Heat capacity, $C_P = (M_0)S$

where S is the specific heat capacity

For H_2 as well as for N_2,

$C_P - C_V = R$

$\therefore\ (M_0)S_P - (M_0)S_V = R$ or $S_P - S_V = \dfrac{R}{M_0}$

For H_2 gas, $S_P - S_V = \dfrac{R}{2} = a$

For N_2 gas, $S_P - S_V = \dfrac{R}{28} = b$

So, $\dfrac{a}{b} = \dfrac{R/2}{R/28} = 14 \Rightarrow a = 14b$

Solution Senders of Physics Musing

SET-74

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CLASS-XI

BRUSH UP for NEET/JEE 2020

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Unit 4 System of particles and Rotational Motion

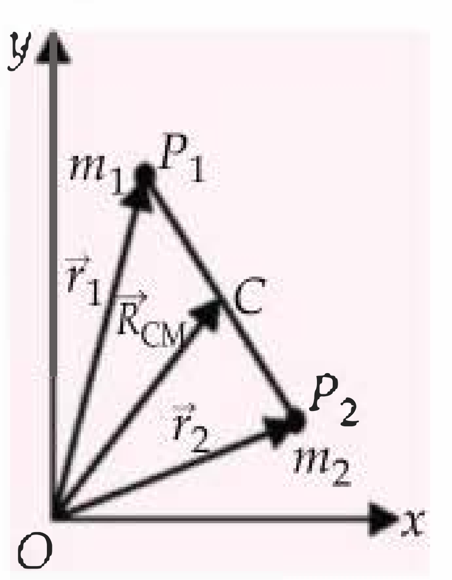
CENTRE OF MASS

The centre of mass of a system of particles is the point that moves as though

- All of the system mass is concentrated there and
- All external forces are applied there.

Centre of mass of a two particle system

- Consider a system of two particles P_1 and P_2 of masses m_1 and m_2 respectively. Let $\vec{r}_1$ and $\vec{r}_2$ be their position vectors with respect to the origin O, as shown in Fig. The position vector $\vec{R}_{CM}$ of the centre of mass C of the two particle system is given by



$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

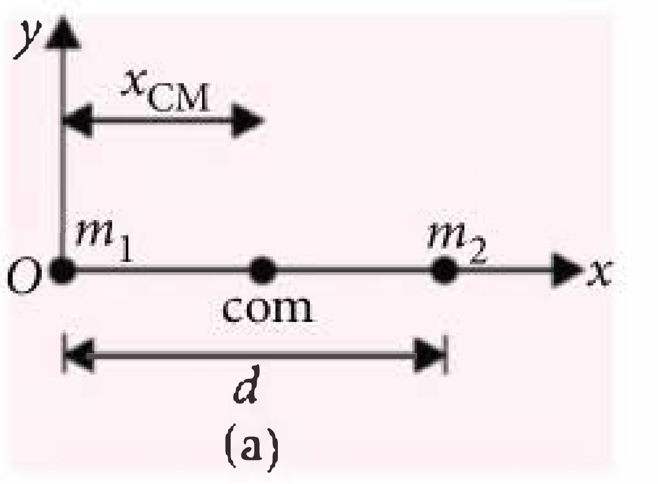
If $m_1 = m_2 = m$(say), then, $\vec{R}_{CM} = \frac{\vec{r}_1 + \vec{r}_2}{2}$

Thus the centre of mass of two equal masses lies exactly at the centre of the line joining the two masses.

If (x_1, y_1) and (x_2, y_2) are the coordinates of the locations of the two particles, then the coordinates of their centre of mass are given by

$$X_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} \text{ and } Y_{CM} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$

- In figure (a), two particles of masses m_1 and m_2 are separated by distance d. The position of the centre of mass (CM) of these two particles is given by



(a)

$$x_{CM} = \frac{m_2}{m_1 + m_2}d$$

Centre of mass of n-particles system

- For a system of n-particles of masses m_1, m_2, m_3, ..., m_n having the position vectors $\vec{r}_1, \vec{r}_2, ..., \vec{r}_n$ respectively with respect to coordinate system, the position of the centre of mass is given by a position vector

$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n}{m_1 + m_2 + ... + m_n} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{M}$$

where, M is the total mass of the system.

The coordinates of centre of mass are given by

$$x_{CM} = \frac{\sum_{i=1}^{n} m_i x_i}{M}\ ;\ y_{CM} = \frac{\sum_{i=1}^{n} m_i y_i}{M}\ ;\ z_{CM} = \frac{\sum_{i=1}^{n} m_i z_i}{M}$$

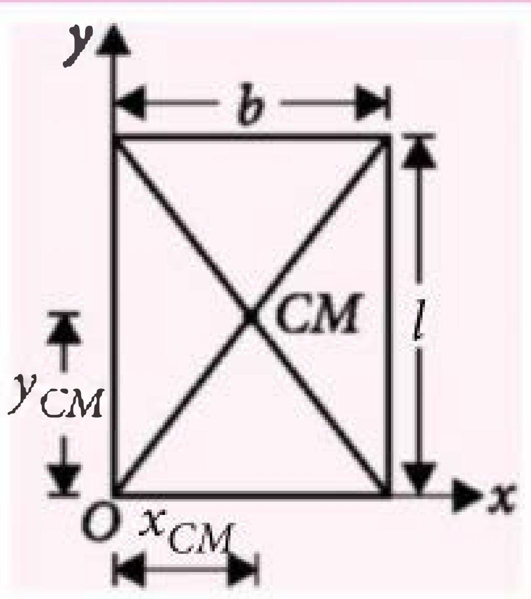
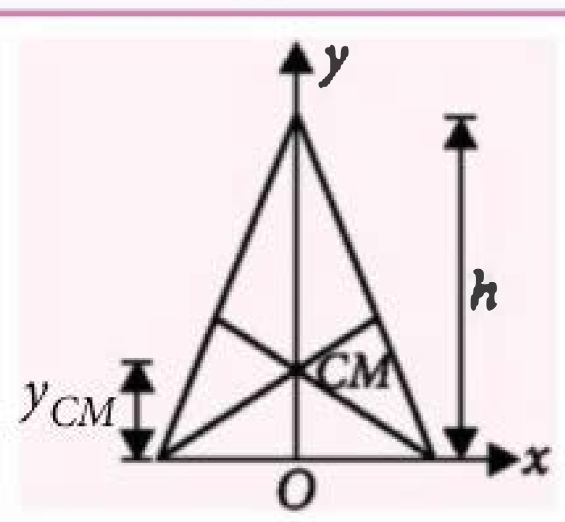
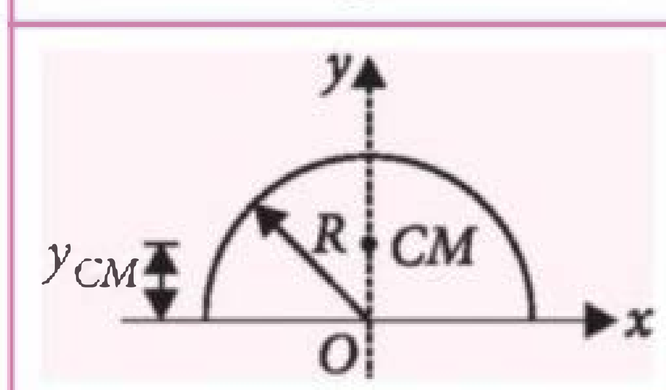
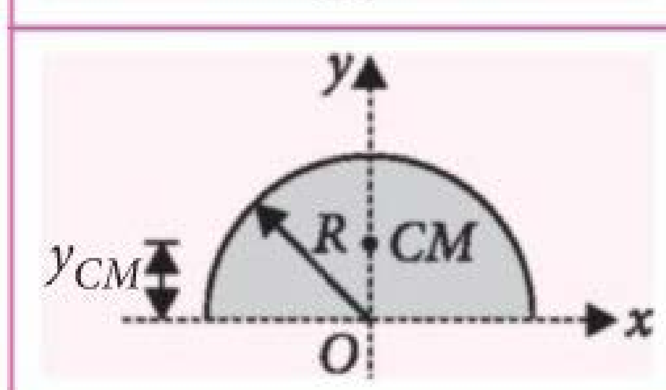
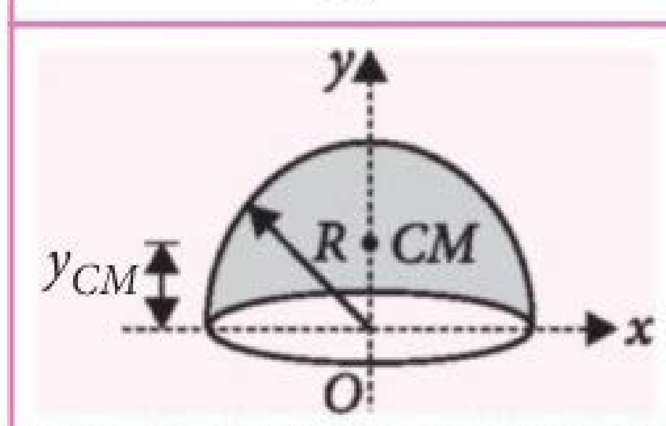
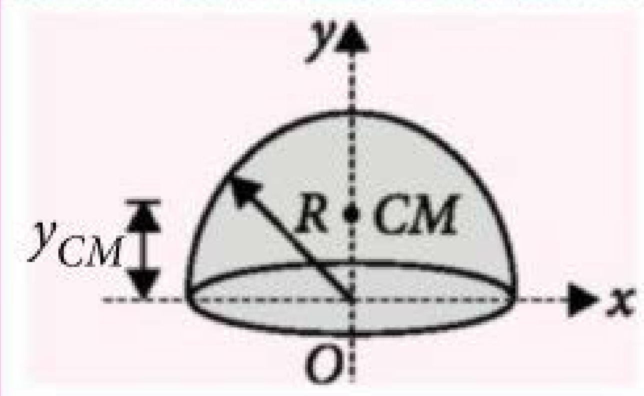
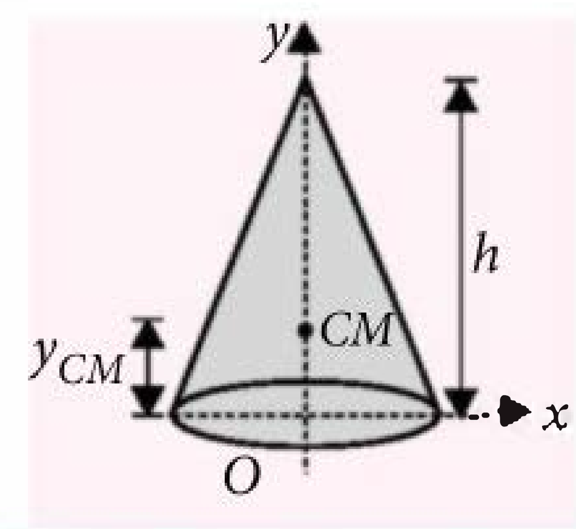
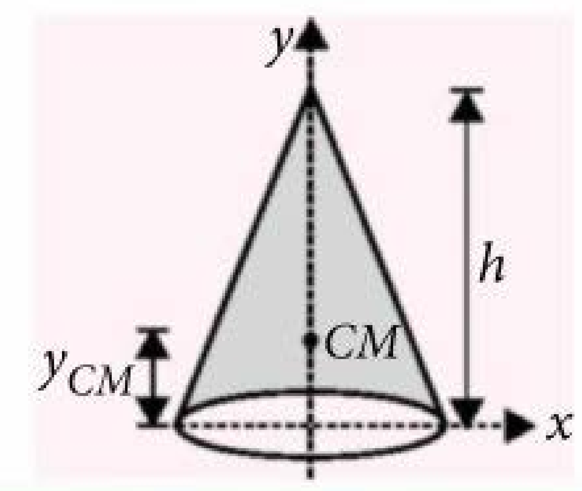
Centre of mass of a rigid body (or continuous distribution of mass)

- The solid bodies contain so many particles (atoms) that we can treat the bodies as a continuous distribution of matter.

- The 'particles' then become differential mass elements dm, the sum become integrals, and the coordinates of the centre of mass are defined as

$$x_{CM} = \frac{1}{M}\int x\,dm\ ;\ y_{CM} = \frac{1}{M}\int y\,dm\ ;\ z_{CM} = \frac{1}{M}\int z\,dm$$

  where M is the mass of the body.
- The position of the centre of mass of a system is independent of the choice of coordinate system.
- The position of the centre of mass depends on the shape of the body and the distribution of its mass. Hence it may lie within or outside the material of the body.
- In symmetrical bodies in which the distribution of mass is homogeneous, the centre of mass coincides with the centre of symmetry or geometrical centre.
- **Centre of mass of some well known symmetric rigid bodies :**

Position of Centre of Mass of different bodies	
	Rectangular plate $x_{CM} = \frac{b}{2}$, $y_{CM} = \frac{l}{2}$
	Triangular plate At the centroid, $y_{CM} = \frac{h}{3}$, $x_{CM} = 0$
	Semi-circular ring $y_{CM} = \frac{2R}{\pi}$, $x_{CM} = 0$
	Semi-circular disc $y_{CM} = \frac{4R}{3\pi}$, $x_{CM} = 0$
	Hemispherical shell $y_{CM} = \frac{R}{2}$, $x_{CM} = 0$
	Solid hemisphere $y_{CM} = \frac{3R}{8}$, $x_{CM} = 0$
	Circular cone (solid) $y_{CM} = \frac{h}{4}$, $x_{CM} = 0$
	Circular cone (hollow) $y_{CM} = \frac{h}{3}$, $x_{CM} = 0$

Motion of Centre of Mass

- For a system of particles, position of centre of mass is $\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots\ldots}{m_1 + m_2 + m_3 + \ldots\ldots}$

  So, Velocity of centre of mass

$$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\ldots}{m_1 + m_2 + \ldots\ldots} \qquad \left(\because \frac{d\vec{r}}{dt} = \vec{v}\right)$$

- Acceleration of centre of mass

$$\vec{a}_{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots.}{m_1 + m_2 + \ldots.} \qquad \left(\because \vec{a} = \frac{d\vec{v}}{dt}\right)$$

- Linear momentum of a system of particles is equal to the product of mass of the system with the velocity of its centre of mass.
- If no external force acts on a system the velocity of its centre of mass remains constant, *i.e.*, velocity of centre of mass is unaffected by internal forces.

  From Newton's second law $\vec{F}_{ext} = \frac{d(M\vec{v}_{CM})}{dt}$

  If $\vec{F}_{ext} = 0$ then $\vec{v}_{CM} = \text{constant}$.

ANGULAR VELOCITY AND ANGULAR ACCELERATION

Angular Velocity

- It is defined as the time rate of change of angular displacement and is given by $\omega = \frac{d\theta}{dt}$.

- Angular velocity is directed along the axis of rotation. Angular velocity is a vector quantity. Its SI unit is rad s^{-1} and its dimensional formula is $[M^0L^0T^{-1}]$.

Relationship between Linear velocity and Angular Velocity

- The linear velocity of a particle of a rigid body rotating about a fixed axis is given by $\vec{v} = \vec{\omega} \times \vec{r}$ where $\vec{r}$ is the position vector of the particle with respect to an origin along the fixed axis.
- As in pure translation motion, all particles of the body have the same linear velocity at any instant of time, in pure rotational motion, all particles of the body have the same angular velocity at any instant of time.
- For rotation about a fixed axis, the direction of the angular velocity ω does not change with time. Its magnitude, however may change from instant to instant. In general rotation, both the magnitude and direction of ω may change from instant to instant.

Angular Acceleration

- It is defined as the time rate of change of angular velocity and is given by, $\alpha = \frac{d\omega}{dt}$
- Angular acceleration is a vector quantity. Its SI unit is rad s^{-2} and its dimensional formula is $[M^0L^0T^{-2}]$

Kinematics Equations of Rotational Motion

- $\omega = \omega_0 + \alpha t$
- $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
- $\omega^2 = \omega_0^2 + 2\alpha\theta$

where the symbols have their usual meanings.

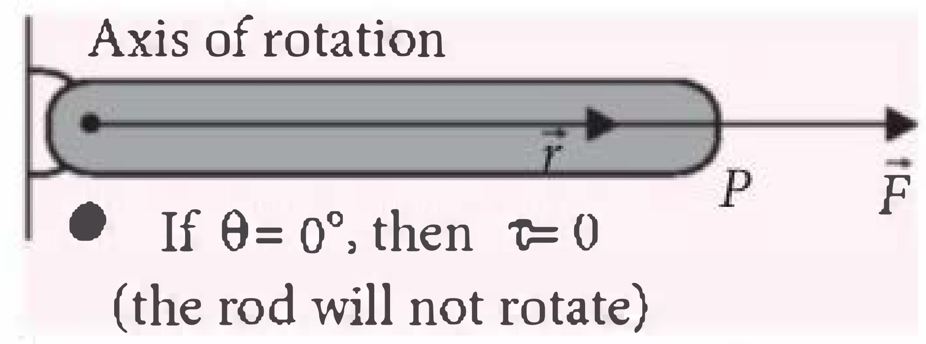
Torque and Angular Momentum

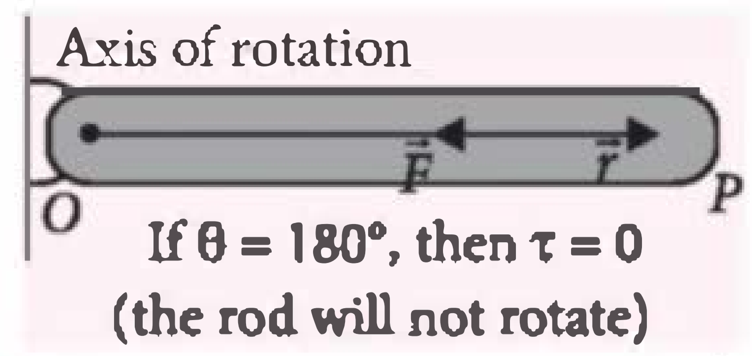
Torque

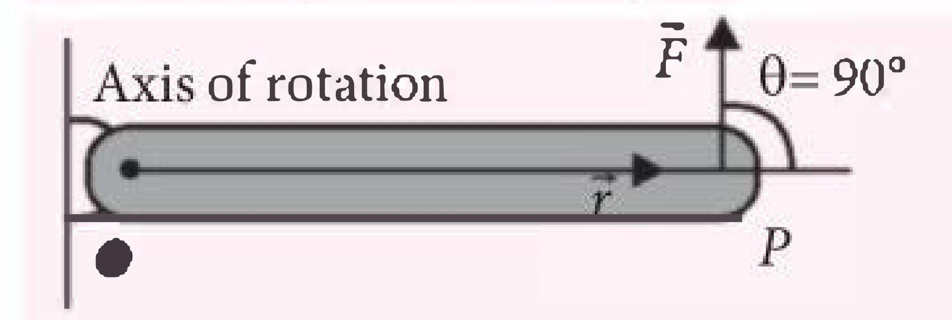
- Torque is the turning effect of a force. If a force acting on an object has a tendency to rotate the body about an axis, the force is said to exert a torque on the body. It is a vector quantity.
- In vector form,
  Torque, $\tau = \vec{r} \times \vec{F}$
  In magnitude, $\tau = r F \sin\theta$.
  Here θ is the angle between $\vec{r}$ and $\vec{F}$.
- Torque has the same dimensions as that of work *i.e.* $[ML^2T^{-2}]$. But work is a scalar quantity whereas torque is a vector quantity.

- By convention, anticlockwise moments are taken as positive and clockwise moments are taken as negative.

Special Cases

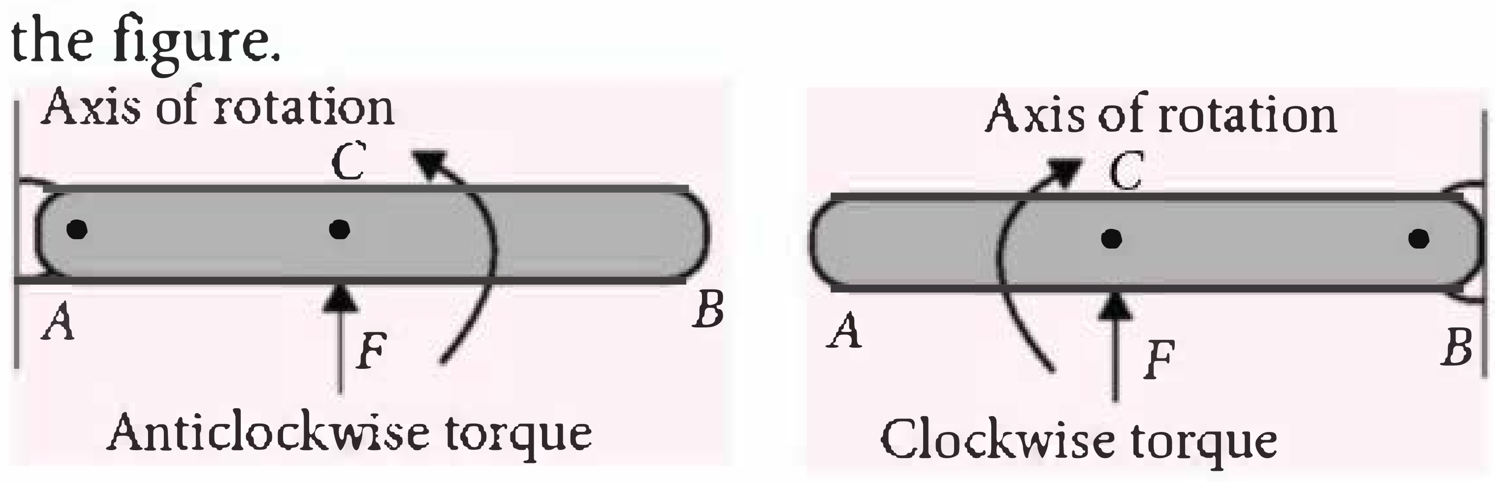
- 

- 

- 

If θ = 90°, then τ = rF (maximum torque)

Note : Same force acting at the same point can produce either anticlockwise or clockwise torque depending upon the location of the axis of rotation as shown in the figure.



Work Done by a Torque and Power

- Work done, W = torque × angular displacement
  $= \tau \times \Delta\theta$
- Power, $P = \frac{dW}{dt} = \tau\frac{d\theta}{dt} = \tau\omega$

Angular Momentum

- Angular momentum of a particle about a given axis is the moment of linear momentum of the particle about that axis. It is denoted by symbol $\vec{L}$.
- Angular momentum $\vec{L} = \vec{r} \times \vec{p}$
  In magnitude, $L = rp\sin\theta$
  where θ is the angle between $\vec{r}$ and $\vec{p}$.
- Angular momentum is a vector quantity. Its SI unit is kg m^2 s^{-1}. Its dimensional formula is $[ML^2T^{-1}]$.

Relationship between Torque and Angular Momentum

Rate of change of angular momentum of a body is equal to the external torque acting upon the body.

Mathematically, $\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$

Law of Conservation of Angular Momentum

If no external torque acts on a system then the total angular momentum is conserved.

i.e., $\vec{\tau}_{ext} = 0$ then $\frac{d\vec{L}}{dt} = 0$ or $\vec{L} = \text{constant}$

Thus, if no external torque acts on a system, total angular momentum of the system remains constant. This is a statement of law of conservation of angular momentum.

Applications of Conservation of Angular Momentum

- A planet revolves around the sun in an elliptical path. When it comes near the sun, the moment of inertia of the planet about the sun decreases. In order to conserve the angular momentum, the angular velocity shall increase. Similarly, when the planet is away from the sun, there will be decrease in the angular velocity.
- An ice-skater or a ballet dancer can increase her angular velocity by folding her arms and bringing the stretched leg close to the other leg. When she stretches her hands and a leg outward, her moment of inertia increases and hence angular speed decreases to conserve angular momentum. When she folds her arms and brings the stretched leg close to the other leg, her moment of inertia decreases and hence angular speed increases.

Equilibrium of a Rigid Body

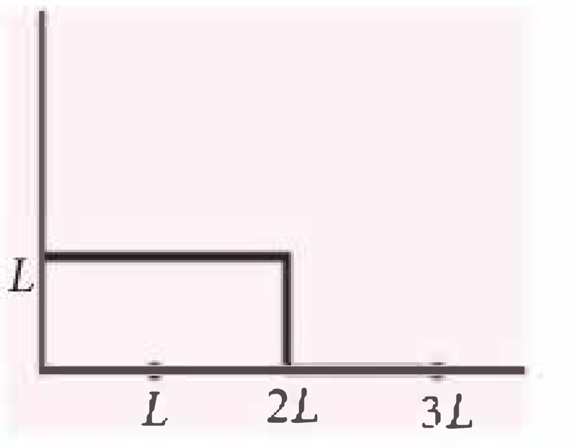
- A body is in rotational equilibrium, if the total external torque on it is zero, *i.e.* $\Sigma\tau_i = 0$.

  $\frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} = \text{constant}$

  i.e., the body rotates with a constant angular velocity.
- **Center of gravity :** A point of an extended body where the weight of the body acts and total gravitational torque on the body is zero, is known as the centre of gravity.

Peep Into Previous Years

1. The position vector of the centre of mass $\vec{r}_{cm}$ of an asymmetric uniform bar of negligible area of cross-section as shown in figure is



(a) $\vec{r}_{cm} = \frac{11}{8}L\hat{x} + \frac{3}{8}L\hat{y}$

(b) $\vec{r}_{cm} = \frac{13}{8}L\hat{x} + \frac{5}{8}L\hat{y}$

(c) $\vec{r}_{cm} = \frac{3}{8}L\hat{x} + \frac{11}{8}L\hat{y}$

(d) $\vec{r}_{cm} = \frac{5}{8}L\hat{x} + \frac{13}{8}L\hat{y}$ **(*JEE Main 2019*)**

2. The moment of the force, $\vec{F} = 4\hat{i} + 5\hat{j} - 6\hat{k}$ at (2, 0, –3), about the point (2, –2, –2), is given by
   (a) $-8\hat{i} - 4\hat{j} - 7\hat{k}$ (b) $-4\hat{i} - \hat{j} - 8\hat{k}$
   (c) $-7\hat{i} - 8\hat{j} - 4\hat{k}$ (d) $-7\hat{i} - 4\hat{j} - 8\hat{k}$
   (*NEET 2019*)

Moment of Inertia

- The moment of inertia of a rigid body about a fixed axis is defined as the sum of the products of the masses of the particles constituting the body and the squares of their respective distances from the axis of rotation.
- The dimensional formula of moment of inertia is $[ML^2T^0]$. The SI unit of moment of inertia is kg m^2 and its CGS unit is g cm^2.
- Momentum of inertia is the rotational analogue of mass in linear motion.

  The moment of inertia of a body depends on
  - Mass of the body
  - Size and shape of the body
  - Distribution of mass about the axis of rotation.
  - Position and orientation of the axis of rotation with respect to the body.

Radius of Gyration

- The radius of gyration of a body about its axis of rotation may be defined as the distance from the axis of rotation at which, if the whole mass of the body were concentrated, its moment of inertia about the given axis would be the same as with the actual distribution of mass. It is denoted by symbol k.
- The relation between moment of inertia I and radius of gyration k is

  $I = Mk^2$ or $k = \sqrt{\frac{I}{M}}$
- Radius of gyration of a body about an axis of rotation may also be defined as the root mean square distance of its particles from the axis of rotation.

i.e., $k = \sqrt{\dfrac{r_1^2 + r_2^2 + ... + r_n^2}{n}}$

The SI unit of radius of gyration is m.

Theorem of Perpendicular Axes

- It states that the moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of the moments of inertia of the lamina about any two mutually perpendicular axes in its plane and intersecting each other at the point, where the perpendicular axes pass through the lamina. Mathematically,

  $I_z = I_x + I_y$

  where x-and y-axes lie in the plane of the lamina and z-axis is perpendicular to its plane and passes through the point of intersection of x and y-axes.

Theorem of Parallel Axes

- It states that the moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes. Mathematically, $I = I_{CM} + Md^2$

  where I_{CM} is the moment of inertia of the body about an axis passing through its centre of mass and d is the perpendicular distance between the two parallel axes.

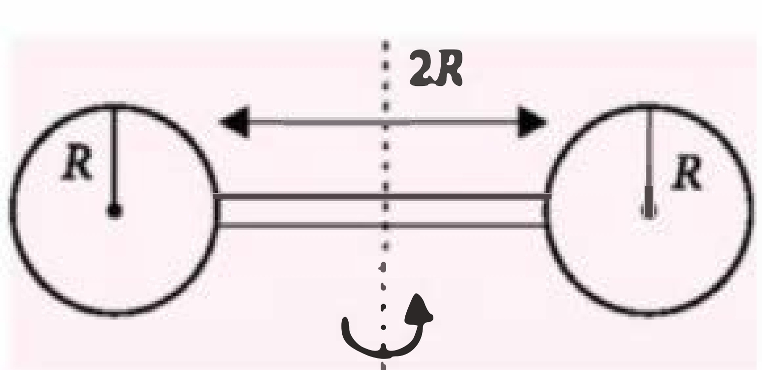
Moment of inertia and radius of gyration of some regular bodies about specific axis is given below :

S.No.	Body	Axis of rotation	Moment of inertia (I)	Radius of gyration (K)
1.	Uniform circular ring of mass M and radius R	(i) about an axis passing through the centre and perpendicular to its plane	MR^2	R
		(ii) about a diameter	$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$
		(iii) about a tangent in its own plane	$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$
		(iv) about a tangent perpendicular to its plane	$2MR^2$	$R\sqrt{2}$
2.	Uniform circular disc of mass M and radius R	(i) about an axis passing through its centre and perpendicular to its plane	$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$
		(ii) about a diameter	$\frac{1}{4}MR^2$	$R/2$
		(iii) about a tangent in its own plane	$\frac{5}{4}MR^2$	$\sqrt{5}\frac{R}{2}$
		(iv) about a tangent perpendicular to its own plane	$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$
3.	Solid sphere of radius R and mass M	(i) about its diameter	$\frac{2}{5}MR^2$	$\sqrt{\frac{2}{5}}R$
		(ii) about a tangential axis	$\frac{7}{5}MR^2$	$\sqrt{\frac{7}{5}}R$
4.	Hollow sphere of radius R and mass M	(i) about its diameter	$\frac{2}{3}MR^2$	$\sqrt{\frac{2}{3}}R$
		(ii) about a tangential axis	$\frac{5}{3}MR^2$	$\sqrt{\frac{5}{3}}R$

5.	Solid cylinder of length l, radius R and mass M	(i) about its own axis	$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$
		(ii) about an axis passing through centre of mass and perpendicular to its own axis	$M\left[\frac{l^2}{12}+\frac{R^2}{4}\right]$	$\sqrt{\frac{l^2}{12}+\frac{R^2}{4}}$
		(iii) about the diameter of one of the faces of cylinder	$M\left[\frac{l^2}{3}+\frac{R^2}{4}\right]$	$\sqrt{\frac{l^2}{3}+\frac{R^2}{4}}$
6.	Hollow cylinder of mass M, length l and radius R	(i) about its own axis	MR^2	R
		(ii) about an axis passing through its centre of mass and perpendicular to its own axis	$M\left(\frac{R^2}{2}+\frac{l^2}{12}\right)$	$\sqrt{\frac{R^2}{2}+\frac{l^2}{12}}$
7.	Thin rod of length L and Mass M	(i) about an axis passing through the centre of mass and perpendicular to the rod	$\frac{ML^2}{12}$	$\frac{L}{\sqrt{12}}$
		(ii) about an axis passing through one end and perpendicular to rod	$\frac{ML^2}{3}$	$\frac{L}{\sqrt{3}}$
8.	Rectangular lamina of length l breadth b and mass M	about an axis passing through its centre of mass and perpendicular to plane	$M\left[\frac{l^2+b^2}{12}\right]$	$\sqrt{\frac{l^2+b^2}{12}}$
9.	Uniform solid cone of radius R, and mass M height h	about an axis through its centre of mass and joining its vertex to centre of base	$\frac{3}{10}MR^2$	$R\sqrt{\frac{3}{10}}$
10.	Parallelopiped of length l, breadth b and height h, mass M	about its central axis	$M\left(\frac{l^2+b^2}{12}\right)$	$\sqrt{\frac{l^2+b^2}{12}}$

Peep Into Previous Years

3. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length $2R$ and mass M(see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is



(a) $\frac{209}{15}MR^2$ (b) $\frac{137}{15}MR^2$
(c) $\frac{152}{15}MR^2$ (d) $\frac{17}{15}MR^2$

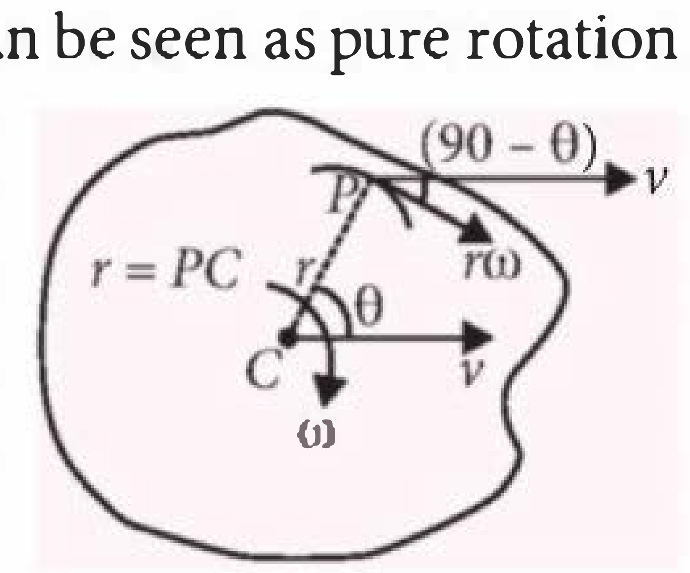
(JEE Main 2019)

4. A solid cylinder of mass 2 kg and radius 4 cm is rotating about its axis at the rate of 3 rpm. The torque required to stop it after 2π revolutions is
(a) 2×10^6 N m (b) 2×10^{-6} N m
(c) 2×10^{-3} N m (d) 12×10^{-4} N m

(NEET 2019)

Rolling

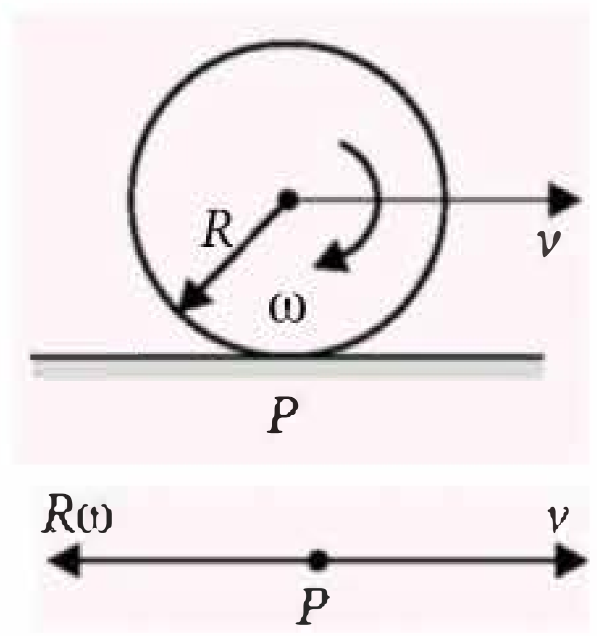
- Rolling is a special phenomenon and it can be understood from two perspectives. Firstly, it can be seen as a special combination of rotation and translation. Secondly, it can be seen as pure rotation about its point of contact.
- Here, velocity of point P is the vector sum of two terms v and $r\omega$. Here v is common for all points, while $r\omega$ is different for different points, as r is different.



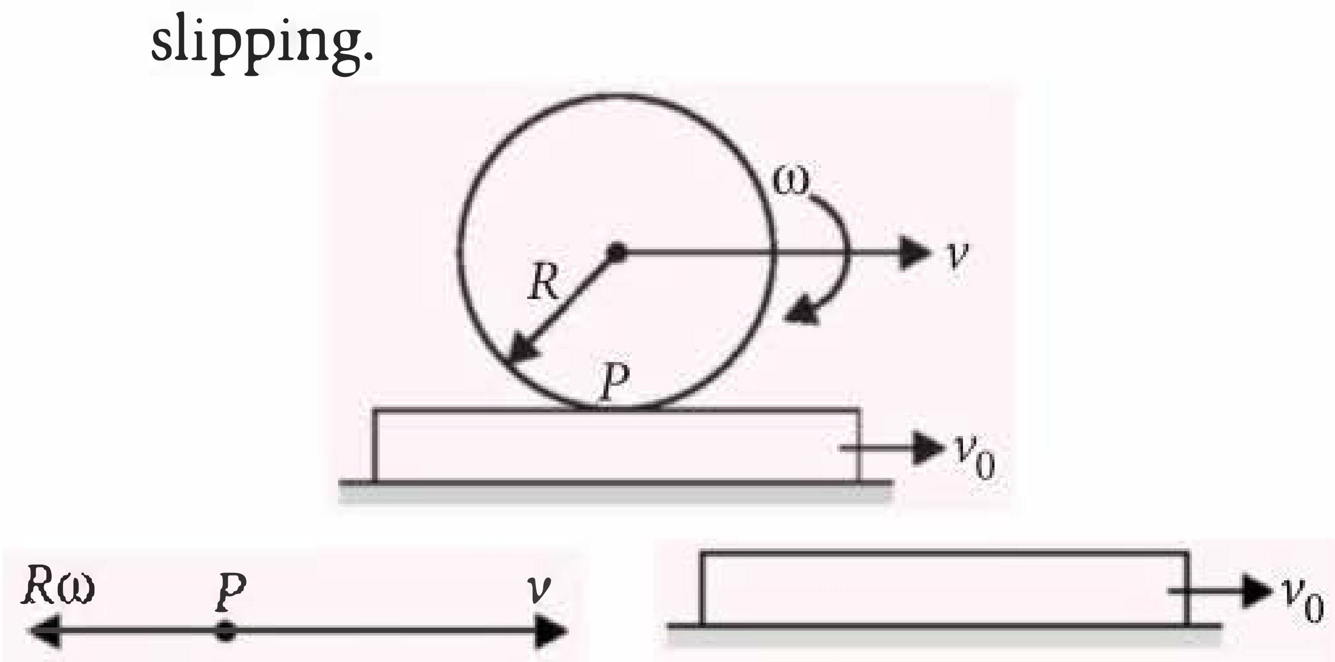
$$v_p = \sqrt{v^2 + (r\omega)^2 + 2(v)(r\omega)\cos(90° - \theta)}$$
$$= \sqrt{v^2 + r^2\omega^2 + 2vr\omega\sin\theta}$$

- Uniform pure rolling : In which v and ω remain constant.

  Condition of pure rolling is $v = R\omega$. In this case bottommost point of the spherical body is at rest. It has no slipping with its contact point on ground. Because ground point is also at rest.



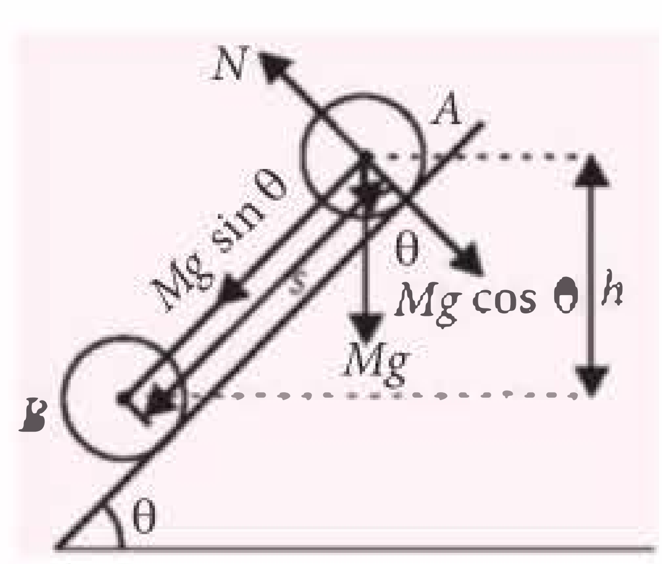
  - If $v > R\omega$, then net velocity of point P is in the direction of v. This is called forward slipping.
  - If $v < R\omega$, then net velocity of point P is in opposite direction of v. This is called backward slipping.



  - If a spherical body is rolling over a plank, condition for no slipping between spherical body and plank is, $v - R\omega = v_0$

Motion of a Body Rolling without Slipping Down an Inclined Plane

- When a rigid body rolls down an inclined plane without slipping, it has a motion of translation as well as of rotation. As it rolls down, it suffers a vertical fall and therefore losses potential energy. At the same time it acquires linear and angular speeds and hence gains kinetic energy of translation and that of rotation. If there is no loss of energy through friction etc., the loss in gravitational potential energy is equal to the gain in kinetic energy.



- Consider a plane inclined at an angle θ to the horizontal and M be the mass and R the radius of that circular section of the body which rolls on the plane.
- Let initially the body be at point A and at rest and after some time it reaches B (figure.).
- In moving from point A to B, the body travels a vertical distance h given by $h = s\sin\theta$, and hence the loss of gravitational potential energy of the body $= Mgh = Mgs\sin\theta$
- Because there is no slipping and it is assumed that there is no dissipation of energy in any other way, the loss in potential energy must be equal to the gain in kinetic energy of the rolling body. Thus,

  $$Mgs\sin\theta = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

  Where I is moment of inertia of the body about an axis passing through its centre and perpendicular to its end faces. Now above equation can be written as

  $$Mgs\sin\theta = \frac{1}{2}Mv^2 + \frac{1}{2}I\left(\frac{v^2}{R^2}\right) \quad \text{...(i)}$$

  On solving,

  $$v = \sqrt{\frac{2Mgh}{M + I/R^2}} = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$$

- Differentiating equation (i), w.r.t. time, we get acceleration of rolling body,

  $$a = \frac{Mg\sin\theta}{M + I/R^2} = \frac{g\sin\theta}{1 + \frac{k^2}{R^2}}$$

- Time of descent of a body rolling down an inclined plane, from equation $s = ut + \frac{1}{2}at^2$ will be,

  $$t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{gM}\left(M + \frac{I}{R^2}\right)} = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1 + \frac{k^2}{R^2}\right)}$$

Analogy between Translational Motion and Rotational Motion

The analogy between quantities that describe linear motion and the corresponding quantities that describe rotational motion is as shown in the table.

	Linear motion	Rotational motion about a fixed axis
1.	Displacement, x	Angular displacement, θ
2.	Velocity, $v = \frac{dx}{dt}$	Angular velocity, $\omega = \frac{d\theta}{dt}$

3.	Acceleration $a = \frac{dv}{dt}$	Angular acceleration $\alpha = \frac{d\omega}{dt}$
4.	Mass, M	Moment of inertia, I
5.	Force, $F = Ma$	Torque, $\tau = I\alpha$
6.	Work done, $W = Fs$	Work done, $W = \tau\theta$
7.	Translational kinetic energy, $K_T = \frac{Mv^2}{2}$	Rotational kinetic energy, $K_R = \frac{I\omega^2}{2}$
8.	Power, $P = Fv$	Power, $P = \tau\omega$
9.	Linear momentum, $p = Mv$	Angular momentum, $L = I\omega$
10.	Equations of translational motion $v = u + at,$ $s = ut + \frac{1}{2}at^2$ $v^2 - u^2 = 2as$ where the symbols have their usual meaning.	Equations of rotational motion $\omega = \omega_0 + \alpha t$ $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ $\omega^2 - \omega_0^2 = 2\alpha\theta$ where the symbols have their usual meaning.

Peep Into Previous Years

5. Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$, are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$, about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(E_f - E_i)$ is

(a) $-\frac{I_1\omega_1^2}{24}$ (b) $\frac{3}{8}I_1\omega_1^2$

(c) $\frac{I_1\omega_1^2}{6}$ (d) $-\frac{I_1\omega_1^2}{12}$

(JEE Main 2019)

6. A solid sphere is in rolling motion. In rolling motion a body possesses translational kinetic energy (K_t) as well as rotational kinetic energy (K_r) simultaneously. The ratio $K_t : (K_t + K_r)$ for the sphere is

(a) 7 : 10 (b) 5 : 7

(c) 10 : 7 (d) 2 : 5 *(NEET 2018)*

Points For Extra Scoring

- A point mass is tied to one end of a string which is wound around a solid body, then. When the mass is released, it falls vertically downwards. Downward acceleration of point mass,

$$a = \frac{g}{1 + \frac{I}{mR^2}}$$

Tension is string, $T = mg\left[\frac{I}{I + mR^2}\right]$

Velocity of point mass, $v = \sqrt{\frac{2gh}{1 + \frac{I}{mR^2}}}$

Angular velocity of rigid body, $\omega = \sqrt{\frac{2mgh}{I + mR^2}}$

where m is the mass of point mass, R is the radius of the rigid body and I is the moment of inertia of the rigid body.

- Moment of Inertia of annular disc through an axis

Passing through the centre and perpendicular to the plane

$$I = \frac{M}{2}[R_1^2 + R_2^2]$$

Passing through its diameter.

$I = \frac{M}{4}[R_1^2 + R_2^2]$, where R_1 and R_2 are the inner and outer radius respectively.

- Moment of inertia of an equilateral triangular lamina with side a passing through its centre of mass and perpendicular to the plane

$$I = \frac{Ma^2}{6}$$

Answer Key For Peep Into Previous Years

1.	(b)	2.	(d)	3.	(b)	4.	(b)	5.	(a)	6.	(b)

WRAP it up!

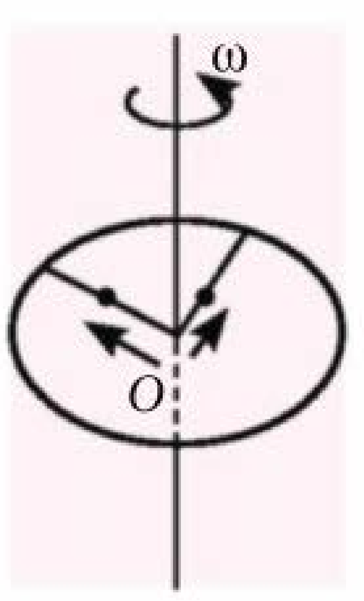
1. The centre of mass of a system of three particles of masses 1 g, 2 g and 3 g is taken as the origin of a coordinate system. The position vector of a fourth particle of mass 4 g such that the centre of mass of the four particle system lies at the point (1, 2, 3) is $\alpha(\hat{i}+2\hat{j}+3\hat{k})$, where α is a constant. The value of α is
(a) 10/3 (b) 5/2 (c) 1/2 (d) 2/5

2. The rotor's velocity of a helicopter engine changes from 330 revmin^{-1} to 110 rev min^{-1} in 2 minutes. How long does the rotor blades take to stop?
(a) 3 min (b) 4 min (c) 5 min (d) 6 min

3. From a circular ring of mass M and radius R, an arc corresponding to a 90° sector is removed. The moment of inertia of the remaining part of the ring about an axis passing through the centre of the ring and perpendicular to the plane of the ring is k times of MR^2. Then the value of k is
(a) 3/4 (b) 7/8 (c) 1/4 (d) 1

4. What constant force, tangential to the equator should be applied to the earth to stop its rotation in one day?
(a) 1.3×10^{22} N (b) 8.26×10^{28} N
(c) 1.3×10^{23} N (d) None of these.

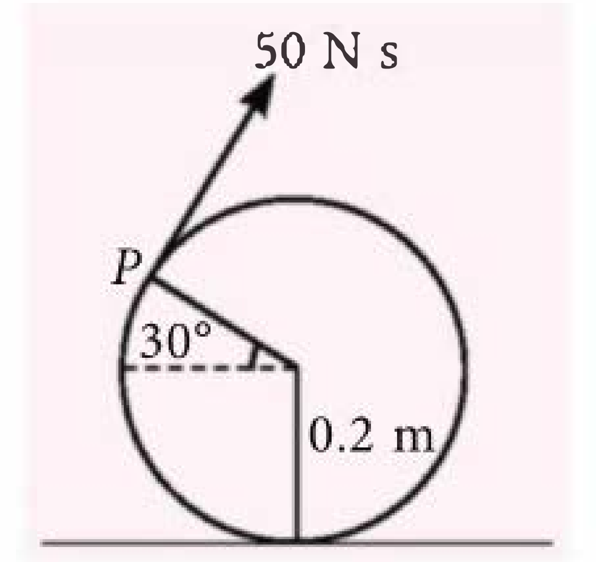
5. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}R$ from O. At this instant the distance of the other mass from O is



(a) $\frac{2}{5}R$ (b) $\frac{1}{3}R$ (c) $\frac{3}{5}R$ (d) $\frac{4}{5}R$

6. A rotating wheel changes angular speed from 1800 rpm to 3000 rpm in 20 s. What is the angular acceleration assuming it to be uniform?
(a) 6π rad s^{-2} (b) 9π rad s^{-2}
(c) 2π rad s^{-2} (d) 4π rad s^{-2}

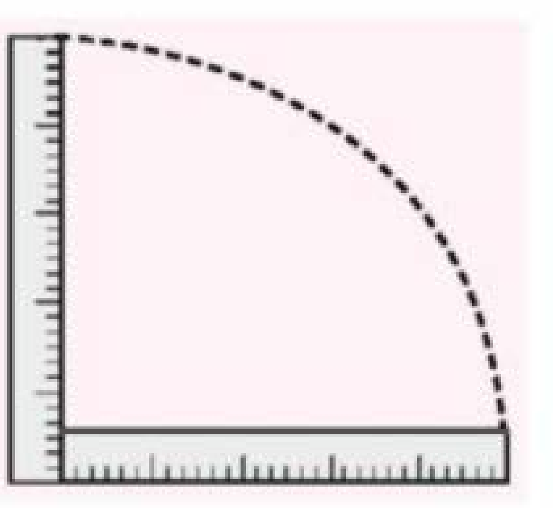
7. A solid ball of radius 0.2 m and mass 1 kg is given an instantaneous impulse of 50 N s at point P as shown in the figure. Find the number of rotations made by the ball about its diameter before hitting the ground. The ball is kept on smooth surface initially.



(a) 1721 (b) 861 (c) 497 (d) 540

8. The centre of mass of a non-uniform rod of length L and mass per unit length $\lambda=\frac{Kx^2}{L}$, where K is a constant and x is the distance from one end, is
(a) $\frac{3L}{4}$ (b) $\frac{L}{8}$ (c) $\frac{K}{L}$ (d) $\frac{3K}{L}$

9. A metre stick is held vertically with one end on the floor and is then allowed to fall. If the end touching the floor is not allowed to slip, the other end will hit the ground with a velocity of (g = 9.8 m s^{-2})



(a) 3.2 m s^{-1} (b) 5.4 m s^{-1}
(c) 7.6 m s^{-1} (d) 9.2 m s^{-1}

10. A coin of mass 10 g rolls along a horizontal table with a velocity of 6 cm s^{-1}. Its total kinetic energy is
(a) 9 μJ (b) 18 μJ (c) 27 μJ (d) 36 μJ

11. A uniform rod AB of mass m and length $2a$ is falling freely without rotation under gravity with AB horizontal. Suddenly the end A is fixed when the speed of the rod is v. The angular speed with which the rod begins to rotate is
(a) $\frac{v}{2a}$ (b) $\frac{4v}{3a}$ (c) $\frac{v}{3a}$ (d) $\frac{3v}{4a}$

12. A solid sphere and a solid cylinder of same masses are rolled down on two inclined planes of heights

h_1 and h_2 respectively. If at the bottom of the plane the two objects have same linear velocities, then the ratio of $h_1 : h_2$ is

(a) 2 : 3 (b) 7 : 5 (c) 14 : 15 (d) 15 : 14

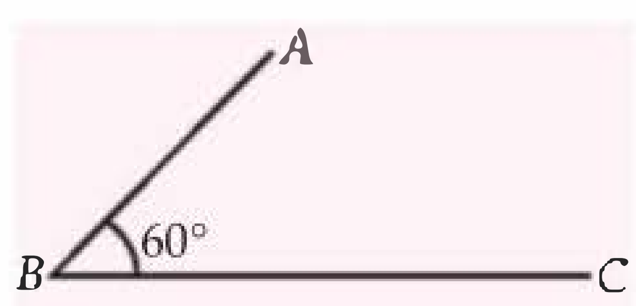
13. A disc is rolling on the inclined plane. What is the ratio of its rotational KE to the total KE?

(a) 1 : 3 (b) 3 : 1 (c) 1 : 2 (d) 2 : 1

14. A disk and a sphere of same radius but different masses roll off on two inclined planes of the same altitude and length. Which one of the two objects reaches the bottom of the plane first?

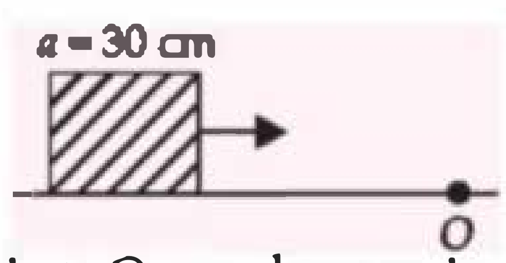
(a) Disk
(b) Sphere
(c) Both reaches at the same time.
(d) Depends on their masses.

15. In the figure shown ABC is a uniform wire. If centre of mass of wire lies vertically below point A, then $\frac{BC}{AB}$ is close to



(a) 1.85 (b) 1.5 (c) 1.37 (d) 3

16. A cubical block of side 30 cm is moving with velocity 2 m s^{-1} on a smooth horizontal surface.



The surface has a bump at a point O as shown in figure. The angular velocity (in rad s^{-1}) of the block immediately after it hits the bump is

(a) 13.3 (b) 5.0 (c) 9.4 (d) 6.7

17. An automobile moves on a road with a speed of 54 km h^{-1}. The radius of its wheels is 0.45 m and the moment of inertia of the wheel about its axis of rotation is 3 kg m^2. If the vehicle is brought to rest in 15 s, the magnitude of average torque transmitted by its brakes to the wheel is

(a) 10.86 kg m^2 s^{-2} (b) 2.86 kg m^2 s^{-2}
(c) 6.66 kg m^2 s^{-2} (d) 8.58 kg m^2 s^{-2}

18. A uniform disc of radius R lies in x-y plane with its centre at origin. Its moment of inertia about the axis $x = 2R$ and $y = 0$ is equal to the moment of inertia about the axis $y = d$ and $z = 0$, where d is equal to

(a) $\frac{4}{3}R$ (b) $\frac{\sqrt{17}}{2}R$ (c) $3R$ (d) $\frac{\sqrt{15}}{2}R$

19. A boat of 90 kg is floating in still water. A boy of mass 30 kg walks from the stern to the bow. The length of the boat is 3 m. Find the distance through which the boat will move.

(a) 0.75 m (b) 0.90 m (c) 1.0 m (d) 1.5 m

20. From a disc of radius R and mass M, a circular hole of diameter R, whose rim passes through the centre is cut. What is the moment of inertia of the remaining part of the disc about a perpendicular axis, passing through the centre?

(a) 11 $MR^2/32$ (b) 9 $MR^2/32$
(c) 15 $MR^2/32$ (d) 13 $MR^2/32$

SOLUTIONS

1. (b) : Let us take the (x, y, z) coordinates of masses 1 g, 2 g, 3 g and 4 g are $(x_1 = 0, y_1 = 0, z_1 = 0)$, $(x_2 = 0, y_2 = 0, z_2 = 0)$
$(x_3 = 0, y_3 = 0, z_3 = 0)$, $(x_4 = \alpha, y_4 = 2\alpha, z_4 = 3\alpha)$

$$\therefore X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

$$X_{CM} = \frac{1\times0 + 2\times0 + 3\times0 + 4\times\alpha}{1+2+3+4}$$

$$1 = \frac{4\alpha}{10} \text{ or } \alpha = \frac{5}{2}$$

The value of α can also be calculated by Y_{CM} and Z_{CM}.

2. (a) : Initial angular velocity of the rotor,

$$\omega_i = 330 \text{ rev min}^{-1} = \frac{2\pi\times330}{60}\text{rad s}^{-1}$$

Final angular velocity of the rotor,

$$\omega_f = 110 \text{ rev min}^{-1} = \frac{2\pi\times110}{60}\text{rad s}^{-1}$$

Angular acceleration of the rotor,

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{\left(\frac{2\pi\times110}{60} - \frac{2\pi\times330}{60}\right)\text{rad s}^{-1}}{2\times60\text{ s}}$$

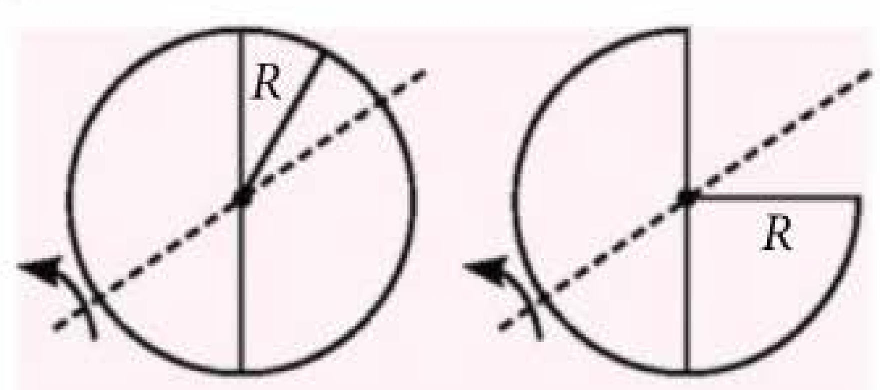
$$= -\frac{2\pi\times220}{2\times60\times60}\text{ rad s}^{-2} = -\frac{2\pi\times110}{60\times60}\text{ rad s}^{-2}$$

when the rotor stops, $\omega_f = 0$

Using $\omega_f = \omega_i + \alpha t$, $0 = \frac{2\pi\times330}{60} - \frac{2\pi\times110}{60\times60}t$

$$t = \frac{2\pi\times330\times60\times60}{2\pi\times110\times60} = 180\text{ s} = 3\text{ min}$$

3. (a) : Moment of inertia of a ring about an axis passing through the centre and perpendicular to the ring is $I = MR^2$



(a) (b)

Mass of the remaining portion of the ring as shown in the figure (b) is $M - \frac{M}{4} = \frac{3M}{4}$

Moment of inertia of the remaining portion of the ring about a given axis is

$I' = \frac{3}{4}MR^2$ ...(i)

Given $I' = kMR^2$ ...(ii)

From (i) and (ii) we get, $k = 3/4$

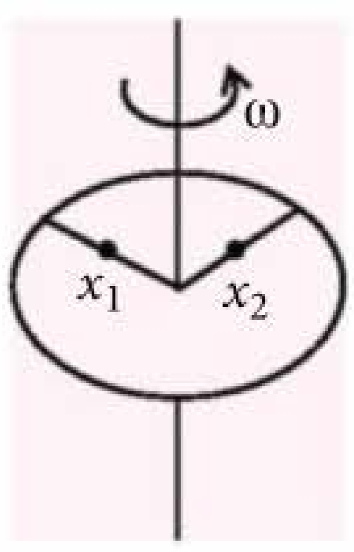
4. (a) : $\omega_1 = 2\pi$ radian per day, $\omega_2 = 0$ and $t = 1$ day

$$\therefore \quad \alpha = \frac{\omega_2 - \omega_1}{t} = \frac{0 - 2\pi}{1} = 2\pi \frac{\text{rad}}{\text{day}^2} = \frac{2\pi}{(86400)^2} \frac{\text{rad}}{\text{s}^2}$$

Torque required to stop the earth $\tau = I\alpha = F \times R$

$$\Rightarrow \quad F = \frac{I\alpha}{R} = \frac{\frac{2}{5}MR^2 \times \alpha}{R} = \frac{2}{5}MR \times \alpha = 1.3 \times 10^{22} \text{ N}$$

5. (d) : By conservation of angular momentum



$$MR^2\omega = \left(MR^2 + \frac{M}{8}\frac{9R^2}{25} + \frac{Md^2}{8}\right)\frac{8\omega}{9}$$

$$R^2 = \left(\frac{200R^2 + 9R^2 + 25d^2}{8 \times 25}\right)\frac{8}{9}$$

$$225R^2 - 209R^2 = 25d^2$$

$$d^2 = \frac{16R^2}{25} \Rightarrow d = \frac{4R}{5}$$

6. (c)

7. (b) : Impulse gives translational velocity

$$u = \frac{\text{Impulse}}{\text{Mass}} \text{ along impulse } = 50 \text{ m s}^{-1}$$

T = time of flight of projectile

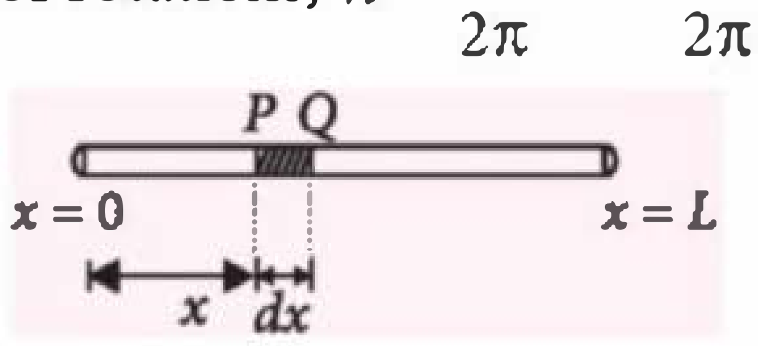
$$= \frac{2u \sin\theta}{g} = \frac{2 \times 50 \times \sin 60^\circ}{10} = 5\sqrt{3} \text{ s}$$

Impulse give angular impulse also

$$\omega = \frac{\text{Impulse} \times R}{I} \quad \text{or} \quad \omega = \frac{\text{Impulse} \times R}{\frac{2}{5}mR^2} = 625 \text{ rad s}^{-1}$$

Number of rotations, $n = \frac{\omega T}{2\pi} = \frac{3125\sqrt{3}}{2\pi} \simeq 861$

8. (a) :



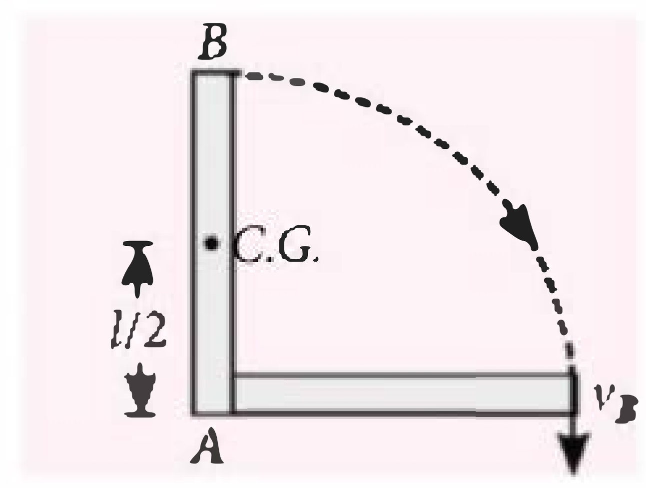
Mass of the element PQ is $dm = \lambda dx = \frac{Kx^2}{L} \cdot dx$

$$\therefore \quad x_{CM} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L \frac{Kx^3}{L}dx}{\int_0^L \frac{Kx^2}{L}dx} = \frac{\left(\frac{L^4}{4}\right)}{\left(\frac{L^3}{3}\right)} = \frac{3L}{4}$$

9. (b): In this process potential energy of the metre stick will be converted into rotational kinetic energy.

Potential energy of metre stick $= mg\left(\frac{l}{2}\right)$

(Because its centre of gravity lies at the middle point of the rod)



Rotational kinetic energy

$E = \frac{1}{2}I\omega^2$

I = Moment of inertia of metre stick about point $A = \frac{ml^2}{3}$

ω = Angular speed of the rod while striking the ground.

v_B = Velocity of end B of metre stick while striking the ground.

By the law of conservation of energy,

$$mg\left(\frac{l}{2}\right) = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{ml^2}{3}\left(\frac{v_B}{l}\right)^2$$

By solving we get, $v_B = \sqrt{3gl} = \sqrt{3 \times 10 \times 1} = 5.4 \text{ m s}^{-1}$.

10. (c) : The total kinetic energy of the coin is

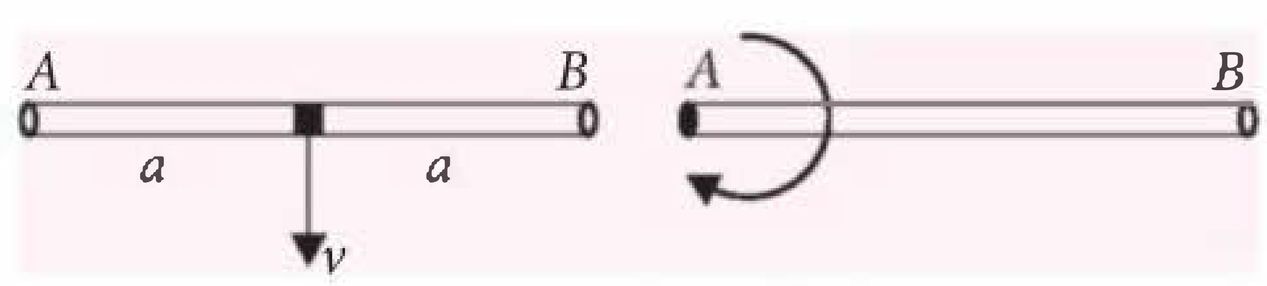
$$K = K_T + K_R = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 \qquad [\because \text{For coin, } I = \frac{1}{2}MR^2]$$

$[\because v = R\omega]$

$$= \frac{3}{4}Mv^2 = 27\mu J.$$

11. (d) :



Angular momentum about A will be conserved, *i.e.*, $L_i = L_f$

or $mva = I\omega$ or $mva = \frac{m(2a)^2}{3}\omega \Rightarrow \omega = \frac{3v}{4a}$

12. (c) : In case of pure rolling of a solid sphere,

$$\frac{K_R}{K_T} = \frac{2}{5}$$

Where K_R = Rotational kinetic energy, K_T = Translational kinetic energy.

Then at the bottom of the plane, $K_R + K_T = mgh_1$

or $\quad K_T = \frac{5}{7}mgh_1 \qquad \left[\text{as } \frac{K_R}{K_T} = \frac{2}{5}\right]$

For solid cylinder, $\frac{K_R}{K_T} = \frac{1}{2}$

As, $K_R + K_T = mgh_2 \quad \therefore \quad K_T = \frac{2}{3}mgh_2$

At the bottom, both have the same linear velocities, *i.e.*, they have the same translational kinetic energies

$$\therefore \quad \frac{5}{7}mgh_1 = \frac{2}{3}mgh_2 \quad \therefore \quad \frac{h_1}{h_2} = \frac{14}{15}$$

13. (a) : $E_{tot} = E_{tran} + E_{rot} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{1}{2}mr^2\times\frac{v^2}{r^2}$$

$$= \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2$$

$$\frac{E_{rot}}{E_{tot}} = \frac{\frac{1}{4}mv^2}{\frac{3}{4}mv^2} = \frac{1}{3} = 1:3$$

14. (b) : Time taken by the body to reach the bottom when it rolls down on an inclined plane without slipping is given by

$$t = \sqrt{\frac{2l\left(1+\frac{k^2}{R^2}\right)}{g\sin\theta}}$$

Since g is constant and l, R and $\sin\theta$ are same for both

$$\therefore \frac{t_d}{t_s} = \frac{\sqrt{1+\frac{k_d^2}{R^2}}}{\sqrt{1+\frac{k_s^2}{R^2}}} = \sqrt{\frac{1+\frac{R^2}{2R^2}}{1+\frac{2R^2}{5R^2}}} \quad \left(\because k_d = \frac{R}{\sqrt{2}}, k_s = \sqrt{\frac{2}{5}}R\right)$$

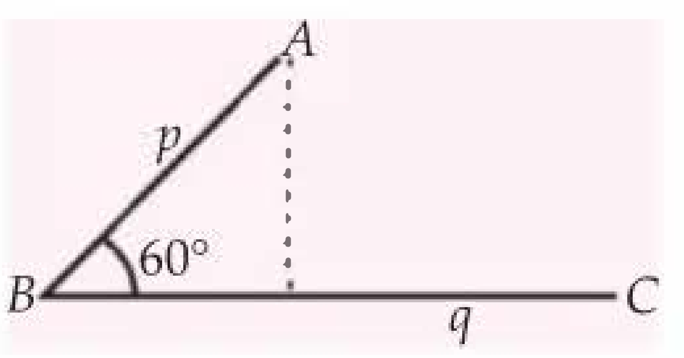
$$= \sqrt{\frac{3}{2}\times\frac{5}{7}} = \sqrt{\frac{15}{14}} \quad \Rightarrow t_d > t_s$$

Hence, the sphere gets to the bottom first.

15. (c) : Let $AB = p$

$BC = q$

λ = linear mass density of the rod



According to question, centre of mass of the rod lies vertically below point A.

$$\therefore \quad X_{CM} = p\cos 60° = \frac{(\lambda q)\left(\frac{q}{2}\right) + (\lambda p)\left(\frac{p}{2}\right)\cos 60°}{\lambda(p+q)}$$

$$\Rightarrow \frac{p}{2} = \frac{\frac{q^2}{2}+\frac{p^2}{4}}{(p+q)} \Rightarrow p^2 + pq = q^2 + \frac{p^2}{2}$$

$$\Rightarrow 1 + \frac{q}{p} = \frac{q^2}{p^2} + \frac{1}{2} \Rightarrow \left(\frac{q}{p}\right)^2 - \frac{q}{p} - \frac{1}{2} = 0$$

$$\frac{q}{p} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)\left(-\frac{1}{2}\right)}}{2\times 1} = \frac{1\pm\sqrt{3}}{2}$$

$\therefore$ Possible value of $\frac{q}{p} = \frac{1+\sqrt{3}}{2} = 1.366 \approx 1.37$

16. (b) : Since no external torque acts on the system, therefore total angular momentum of the system about point O remains constant.

Before hitting, $L_i = mv\frac{a}{2}$

After hitting , $L_f = I\omega$

$\therefore \quad mv\frac{a}{2} = I\omega$ or $\omega = \frac{mva}{2I}$

Here, I = moment of inertia of cube about its edge

$$= m\frac{a^2}{6} + m\left(\frac{\sqrt{2}a}{2}\right)^2 = \frac{ma^2}{6} + \frac{ma^2}{2} = \frac{2ma^2}{3}$$

$$\therefore \quad \omega = \frac{mva\times 3}{2\times 2ma^2} = \frac{3v}{4a} = \frac{3\times 2}{4\times 0.3} = 5 \text{ rad s}^{-1}$$

17. (c) : Here, speed of the automobile,

$v = 54 \text{ km h}^{-1} = 54\times\frac{5}{18} \text{ m s}^{-1} = 15 \text{ m s}^{-1}$

Radius of the wheel of the automobile, $R = 0.45$ m

Moment of inertia of the wheel about its axis of rotation, $I = 3 \text{ kg m}^2$

Time in which the vehicle brought to rest, $t = 15$ s

The initial angular speed of the wheel is

$$\omega_i = \frac{v}{R} = \frac{15 \text{ m s}^{-1}}{0.45 \text{ m}} = \frac{1500}{45}\text{rad s}^{-1} = \frac{100}{3} \text{ rad s}^{-1}$$

and its final angular speed is

$\omega_f = 0 \qquad$ (as the vehicle comes to rest)

$\therefore$ The angular retardation of the wheel is

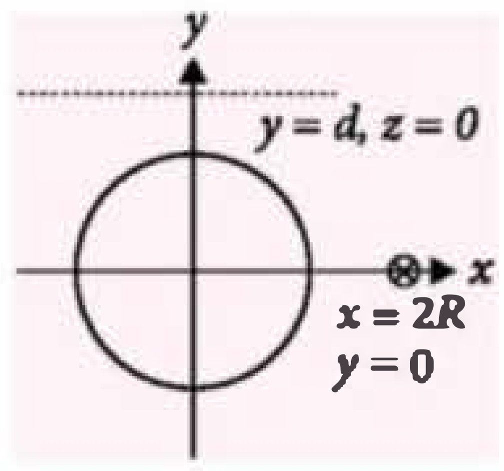
$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{0 - \frac{100}{3}}{15} = -\frac{100}{45} \text{ rad s}^{-2}$$

The magnitude of required torque is

$$\tau = I\alpha = (3\ \text{kg m}^2)\left(\frac{100}{45}\text{rad s}^{-2}\right)$$

$$= \frac{20}{3}\ \text{kg m}^2\text{s}^{-2} = 6.66\ \text{kg m}^2\text{s}^{-2}$$

18. (b): An axis passing through $x = 2R$, $y = 0$ is in ⊗ direction as shown in figure. Moment of inertia about this axis will be



$$I_1 = \frac{1}{2}mR^2 + m(2R)^2 = \frac{9}{2}mR^2 \quad ...(i)$$

Axis passing through $y = d, z = 0$ is shown by dotted line in figure. Moment of inertia about this axis will be

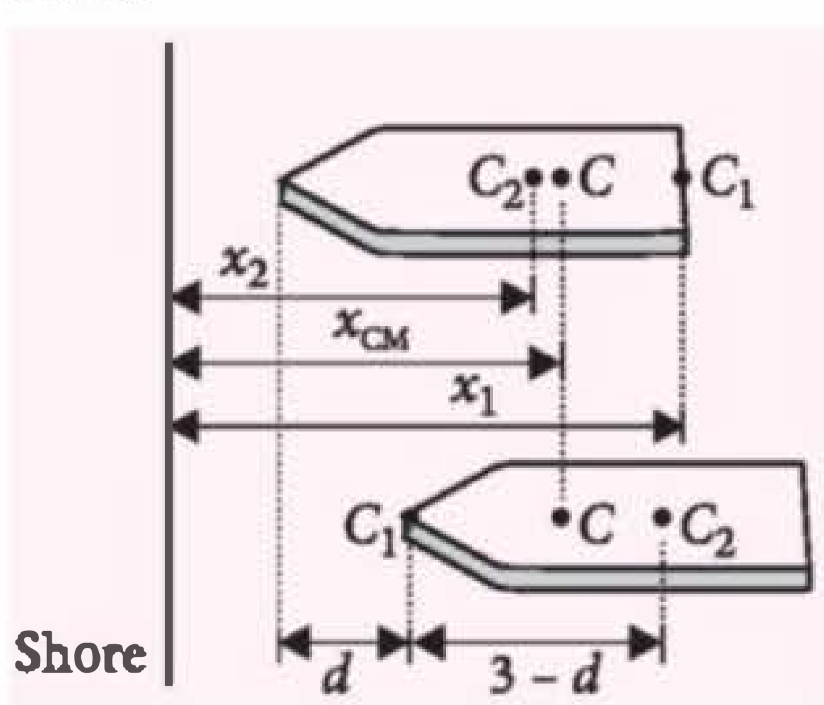
$$I_2 = \frac{1}{4}mR^2 + md^2 \quad ...(ii)$$

By equations (i) and (ii), we get

$$\frac{1}{4}mR^2 + md^2 = \frac{9}{2}mR^2 \text{ or } d = \frac{\sqrt{17}}{2}R$$

19. (a): As shown in figure, let C_1, C_2 and C be the centres of mass of the boy, boat and the system (boy and boat) respectively. Let x_1 and x_2 be the distances of C_1 and C_2 from the shore. Then the centre of mass will be at a distance,

$$x_{CM} = \frac{30x_1 + 90x_2}{30 + 90}$$



As the boy moves from the stern to the bow, the boat moves backward through a distance d so that position of the centre of mass of the system remains unchanged.

$$x'_{CM} = \frac{30[x_1 - (3-d)] + 90(x_2 + d)}{30 + 90}$$

As $x'_{CM} = x_{CM}$

$$\frac{30(x_1 - 3 + d) + 90(x_2 + d)}{120} = \frac{30x_1 + 90x_2}{120}$$

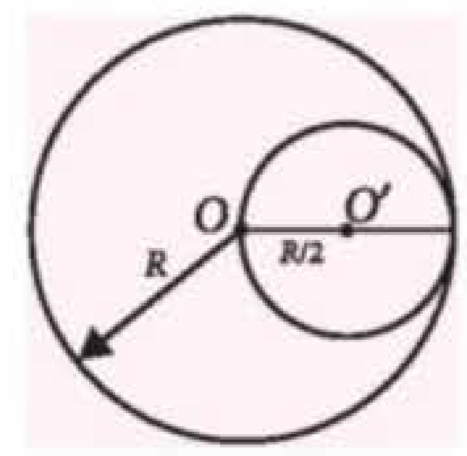
$$\Rightarrow \ = -90 + 30\,d + 90\,d = 0$$

$$\therefore \ d = 0.75\ \text{m}$$

20. (d): Mass per unit area of disc $= \dfrac{M}{\pi R^2}$

Mass of removed portion of disc,

$$M' = \frac{M}{\pi R^2} \times \pi\left(\frac{R}{2}\right)^2 = \frac{M}{4}$$



Moment of inertia of removed portion about an axis passing through centre of disc O and perpendicular to the plane of disc,

$$I'_O = I_{O'} + M'd^2$$

$$= \frac{1}{2} \times \frac{M}{4} \times \left(\frac{R}{2}\right)^2 + \frac{M}{4} \times \left(\frac{R}{2}\right)^2 = \frac{MR^2}{32} + \frac{MR^2}{16} = \frac{3MR^2}{32}$$

When portion of disc would not have been removed, the moment of inertia of complete disc about centre O is

$$I_O = \frac{1}{2}MR^2$$

So, moment of inertia of the disc with removed portion is

$$I = I_O - I'_O = \frac{1}{2}MR^2 - \frac{3MR^2}{32} = \frac{13MR^2}{32}$$

◆◆

EXAM ALERT

NEET 2020 DATE SHEET RELEASED

The entrance exam for the MBBS/BDS courses in India — National Eligibility cum Entrance Test (NEET) for undergraduate courses is going to be held on **May 3, 2020 (Sunday)**. This is the second time the National Testing Agency (NTA) will be conducting the NEET. The application process will begin from December 2 and will close on December 31. Interested candidates can apply at **ntaneet.nic.in**.

All those candidates who have passed class 12 are eligible to appear for the National Eligibility Entrance Test (NEET UG 2020). The exam will be held in the pen-and-paper mode and from **March 27**, the applicants can download the **NEET admit card**. The result for NEET is scheduled to release on **June 4**.

NEET is a three-hour long exam which includes three sections – physics, chemistry and biology. Of the total 180 questions, 90 would be from biology and 45 each from physics and chemistry. Preparation syllabus includes the whole of class 11 and 12 standard NCERT textbooks in the respective subjects.

Every correct answer would get plus four marks and every incorrect answer result in a negative mark. Questions that are not attempted do not have any penalty marks. The National Testing Agency (NTA) has been set-up by the government of India responsibility of NTA to conduct the entrance test. Competitive entrance exams including NEET UG, JEE, NET, etc. were earlier conducted by CBSE. In NEET 2019, record 15 lakh candidates applied and the exam was held on May 5.

WHY MOON MATTERS

Earth's satellite will likely be the base for future interplanetary missions. A presence there is of strategic importance.

Visitors to a village called Khodad, about 80 km north of Pune, can easily spot a series of giant dishes that rise against the sky. These are part of a network of radio telescopes called the Giant Metrewave Radio Telescope (GMRT). Built by the National Centre for Radio Astrophysics (NCRA) in Pune, GMRT looks deep into space and back in time, at events in the universe not easily accessible to optical telescopes.

A set of radio astronomers at NCRA are especially interested in events soon after the Big Bang, with which the universe is supposed to have been born, 13.8 billion years ago. Optical telescope can only look back to about 500 million years after the Big Bang. Radio telescopes alone can peer into the secrets beyond that. But radio telescopes like GMRT are running into a problem on the earth — interference from mobile towers and other electronic equipment.

Sometime in the near future, when the moon opens up for scientists to set up experiments, radio astronomers will be among the first set of scientists to exploit the radio-quiet zones there. A network of radio telescopes on the moon can peer deep into space without electrical interference. With proven expertise in telescope-building, India will be an automatic partner for international projects to build radio telescopes on the moon.

When the GMRT was built 20 years ago, Khodad was a quiet village. Now mobile towers have sprouted everywhere, and radio astronomers at NCRA have had to develop clever ways to weed out the noise. Now no place on the earth is free from signal interference.

The echo of electrical signals from the earth can reach even the near side of the moon, but telescopes on its far side will be completely free from interference. "The far side of the moon is completely shielded from the earth," says Yashwant Gupta, director of NCRA. "Setting up a telescope there is like a dream."

Radio astronomers have been watching the moon keenly for a few decades, hoping to set up their telescopes on the satellite one day. Powerful and sensitive radio telescopes, free from interference, can probe deep questions about the evolution of the universe. Optical telescopes can look back only to a point after the first stars were born. Radio waves carry information about early structures in the universe like the first stars and galaxies.

They are also supposed to carry information about the first atoms in the universe, which may have formed 380,000 years after the Big Bang. But these signals are so feeble — less than a trillionth of the energy of a mobile phone signal — that astronomers need to weed out all noise.

While radio astronomers want to use the moon to figure out events in the early universe, planetary scientists want to explore it to figure out how the solar system was formed, and how the earth evolved into its present state. The composition of the moon and the earth are so similar that it is sometimes hard for a geologist to figure out where a rock or soil sample has come from. But there is a crucial difference.

What Makes the Moon So Attractive

The moon and the earth share a common past, but the moon is frozen in time

Studying the moon tells us how the earth looked like in the past

The moon is a great place to probe deep into space because telescopes will have no interferences from human activity

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The moon is supposed to have been hived off from the earth when both were still hot, some 4.4 billion years ago, by the impact of a body roughly the size of Mars. It is the reason why the earth and the moon are so similar. However, the earth has evolved while the moon is frozen in time.

The earth has plate tectonics that gradually erases history from its surface. The moon died geologically a billion years ago. Or, so we thought. Recently, scientists have noticed some geological activity on the moon, but it is not of the kind that will erase history on the surface. The moon has had large volcanic eruptions like the earth, but the last major set of eruptions happened 3.2 billion years ago.

Lacking an atmosphere, the moon does not have weather, and so the surface remains largely intact. Looking at the moon is like looking at the earth a long time ago. "The moon is the closest image of the ancient earth," says Deepak Dhingra, assistant professor at IIT Kanpur. Dhingra, who had worked on Chandrayaan-1 while at the Physical Research Laboratory (PRL) in Ahmedabad, is a planetary scientist who studies lunar crust evolution.

Our understanding of the moon has improved in recent times. The most important in the last decade was the presence of water on the moon. Scientists have detected a small layer of water all over the moon. Multiple missions, including Chandrayaan-1, have confirmed the presence of water on the poles. Recent research has also indicated that there is water inside the moon.

If there are large amounts of water on the moon, it may become easier to set up a base there. Apart from sustaining life, water can be split to generate hydrogen and oxygen, and they can serve as fuel during the long lunar night (one lunar night lasts about two earth weeks). It also removes the need to carry water from the earth when setting up bases.

The Apollo missions brought back 380 kg of samples, of which 50% was preserved for five decades. This was because NASA knew that technology to analyse the samples will improve in the future, and so didn't want to waste them. These samples were opened and distributed this year to scientists who wanted to study them. "There will be a lot of discoveries soon," says Dhingra.

The Indian moon missions have been driven by a combination of scientific and strategic factors. The moon will be a base for planetary exploration, and being present there will be of strategic value for any country. It is the reason why the US, Europe, Russia, China, India, Japan, South Korea and a few other countries are all planning moon missions in the future.

Scientists will continue to piggyback on many of these technology-led missions. And hope for exciting discoveries over the next few decades.

Courtesy : The Economic Times

CBSE warm-up!

CLASS-XI

Chapterwise Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2019-20.

Series 4

Mechanical Properties of Solids | Mechanical Properties of Fluids

Time Allowed : 3 hours
Maximum Marks : 70

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) Section A : Q. no. 1 to 20 are very short answer-objective questions and carry 1 mark each.
(iii) Section B : Q. no. 21 to 27 are short answer questions and carry 2 marks each.
(iv) Section C : Q. no. 28 to 34 are long answer-I questions and carry 3 marks each.
(v) Section D : Q. no. 35 to 37 are long answer-II questions and carry 5 marks each.
(vi) There is no overall choice in the question paper. However, internal choices are given in the sections.
(vii) Use log tables if necessary, use of calculators is not allowed.

SECTION - A

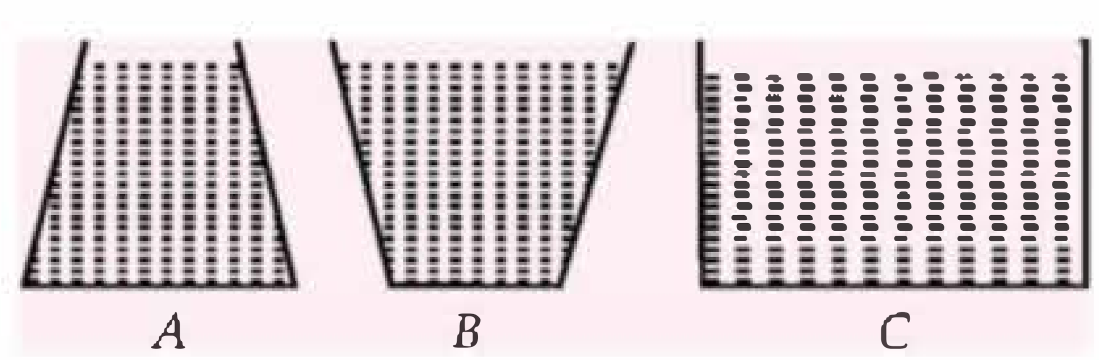
1. A solid sphere falls with a terminal velocity v in air. If it is allowed to fall in vacuum,
(a) terminal velocity of sphere = v
(b) terminal velocity of sphere < v
(c) terminal velocity of sphere > v
(d) sphere never attains terminal velocity.

2. To what height should a cylindrical vessel be filled with a homogeneous liquid to make the force with which the liquid apply pressure on the sides of the vessel equal to the force exerted by the liquid on the bottom of the vessel?
(a) Equal to the radius (b) Less than radius
(c) More than radius (d) Four times of radius

3. The temperature of a wire is doubled. The Young's modulus of elasticity will
(a) also doubled (b) become four times
(c) remains same (d) decreases.

4. The elastic energy stored per unit volume in a stretched wire is
(a) $\frac{1}{2}\frac{\text{stress}}{Y}$ (b) $\frac{1}{2}\frac{(\text{stress})^2}{Y}$
(c) $\frac{1}{2}\frac{(\text{stress})^2}{Y^2}$ (d) $\frac{1}{2}\frac{\text{stress}}{Y^2}$

5. Three vessels A, B and C of different shapes contain water upto the same height as shown in the figure. P_A, P_B and P_C be the pressures exerted by the water at the bottom of the vessels A, B and C respectively. Then



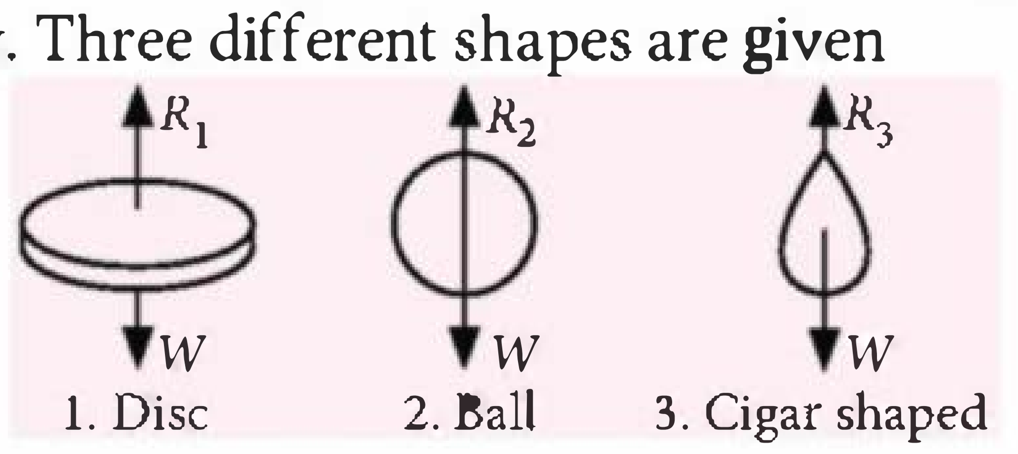
(a) $P_A > P_B > P_C$ (b) $P_B > P_C > P_A$
(c) $P_C > P_B > P_A$ (d) $P_A = P_B = P_C$

6. An ideal fluid flows through a pipe of circular cross-section made of two sections with diameters 2.5 cm and 3.75 cm. The ratio of the velocities in the two pipes is
(a) 9 : 4 (b) 3 : 2
(c) $\sqrt{3}:\sqrt{2}$ (d) $\sqrt{2}:\sqrt{3}$

7. For a constant hydraulic stress on an object, the fractional change in the object's volume ($\Delta V/V$) and its bulk modulus (B) are related as
(a) $\frac{\Delta V}{V} \propto B$ (b) $\frac{\Delta V}{V} \propto \frac{1}{B}$
(c) $\frac{\Delta V}{V} \propto B^2$ (d) $\frac{\Delta V}{V} \propto \frac{1}{B^2}$

8. A wire of length L and radius r is clamped at one end. On stretching the other end of the wire with a force F, the increase in its length is l. If another wire of same material but of length $2L$ and radius $2r$ is stretched with a force $2F$, the increase in its length will be
(a) $l/4$ (b) $l/2$ (c) l (d) $2l$

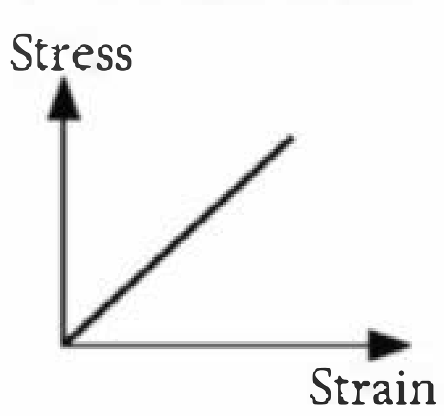
9. When a body falls in air, the resistance of air depends on a greater extent on the shape of the body. Three different shapes are given

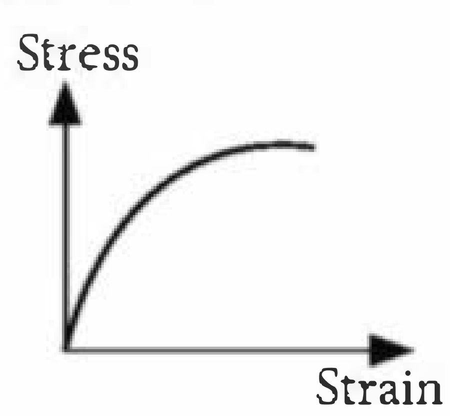


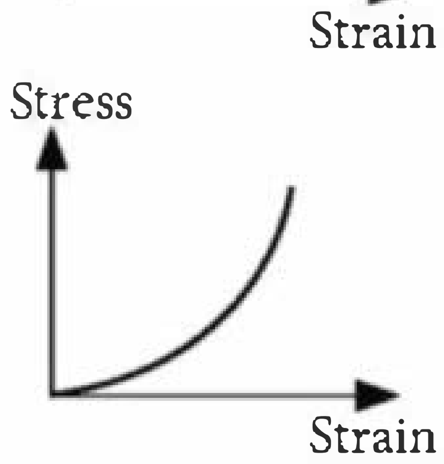
1. Disc 2. Ball 3. Cigar shaped

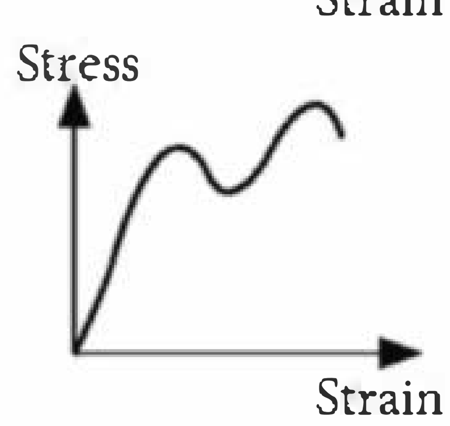
Identify the combination of air resistances, which truly represents the physical situation (The cross-sectional areas are the same).
(a) 1 < 2 < 3 (b) 2 < 3 < 1
(c) 3 < 2 < 1 (d) 3 < 1 < 2

10. Which of the following graphs represents stress-strain variation for elastomers?

(a)


(b)


(c)


(d)


11. Why does a wire get heated when it is bent back and forth?

12. Amorphous solids are not true solids. Why and what are they called then?

13. Why do spring balances show wrong readings after they have been used for a long time?

14. A soft plastic bag weighs the same when empty as when filled with air at atmospheric pressure. Why?

15. Why it is dangerous to stand near the edge of the platform when a fast train is crossing it?

16. Why is it difficult to stop bleeding from a cut in human body at high altitudes ?

17. Two identical springs of copper and steel are equally stretched. On which one, more work will have to be done?

18. Why does velocity increase when water flowing in a broad pipe enters a narrow pipe?

19. Arrange the value of bulk modulus of elasticity of solids, liquids and gases according to their magnitude. Give reason.

20. A small spherical ball of density ρ is gently released in a liquid of density $\sigma (\rho > \sigma)$. Find the initial acceleration of the ball.

SECTION - B

21. Which one of them is more elastic - rubber or steel? Explain.

22. Find the velocity of efflux of water from an orifice near the bottom of a tank in which pressure is 500 g f cm^{-2} above atmosphere.

23. A cable is replaced by another of the same length and material but of twice the diameter. (a) How does this affect its elongation under a given load? (b) Find the maximum load it can now support without exceeding the elastic limit.

OR

The breaking force for a wire is F. What will be the breaking force for (a) two parallel wires of the same size (b) a single wire of double the thickness ?

24. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth $4y$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, what is the value of R?

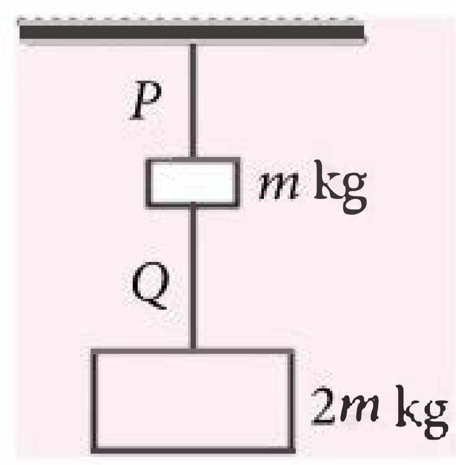
25. Two soap bubbles of different diameters are in contact with a certain portion common to both the bubbles. What will be the shape of the common

boundary as seen from inside the smaller bubble? Support your answer with a neat diagram. Give reason for your answer.

OR

A frame made of metallic wire enclosing a surface area A is covered with a soap film. If the area of the frame of metallic wire is reduced by 50%. What will be the change in the surface energy of the soap film?

26. Two wires P and Q of same diameter are loaded as shown in the figure. The length of wire P is L m and its Young's modulus is Y N m^{-2}, while length of wire Q is twice that of P and its material has Young's modulus half that of P. Compute the ratio of their elongation.



27. The density of a metal at normal pressure is ρ. Its density when it is subjected to an excess pressure P is ρ'. If B is the bulk modulus of the metal, then find the ratio ρ'/ρ.

SECTION-C

28. A wire of length l and radius r has a weight W and the Young's modulus of elasticity Y. It is suspended vertically from a fixed point. Calculate the increase in length of wire produced due to its own weight.

29. Two wires of same length and material but of different radii are suspended from a rigid support. Both carry the same load. Will the stress, strain and extension in them be same or different?

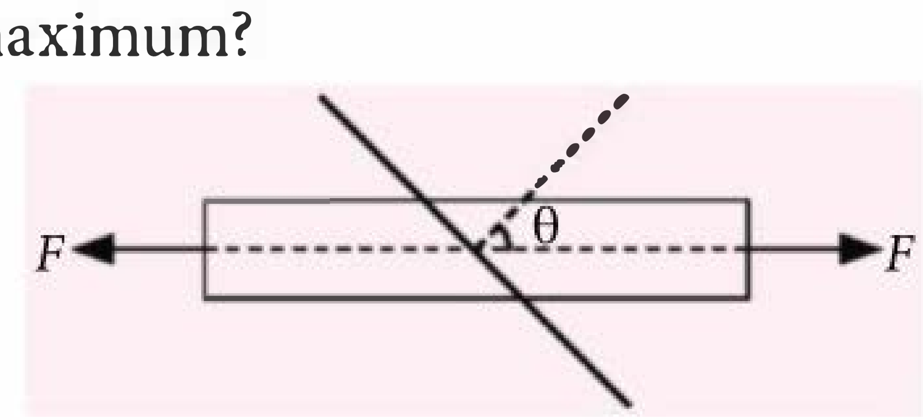
OR

A bar of cross-section A is subjected to equal and opposite tensile forces at its ends. Consider a plane section of the bar whose normal makes an angle θ with the axis of the bar.

(a) What is the tensile stress on this plane?

(b) What is the shearing stress on this plane?

(c) For what value of θ is the tensile stress maximum?



30. A piece of ice with a stone in it floats on water taken in a beaker. When the ice melts completely, the level of water in beaker will increase, decrease or remain unchanged. Explain.

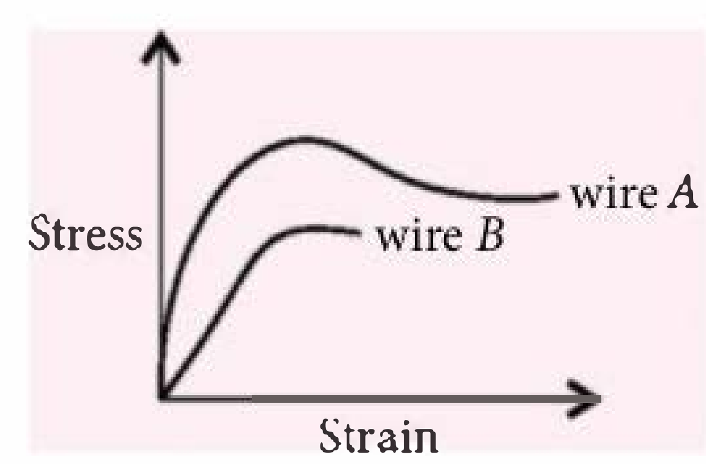
31. Water at a pressure of 4×10^4 N m^{-2} flows at a speed of 2 m s^{-1} through a horizontal pipe of cross-sectional area 0.02 m^2. The cross-sectional area is reduced to 0.01 m^2. What is the pressure in the smaller cross-section of the pipe?

32. A big size balloon of mass M is held stationary in air with the help of a small block of mass $M/2$ tied to it by a light string such that both float in mid air. Describe the motion of the balloon and the block when the string is cut. Support your answer with calculations.

OR

During blood transfusion, the needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein? The density of blood $= 1.06 \times 10^3$ kg m^{-3}.

33. Stress-strain curve for two wires of material A and B are as shown in figure.



(a) Which material is more ductile?

(b) Which material has greater value of Young's modulus?

(c) Which of the two is a stronger material?

(d) Which material is more brittle?

34. Define terminal velocity. Derive an expression for it.

SECTION-D

35. State and prove Bernoulli's theorem.

OR

What is a tube of flow? Obtain a relation between the area of cross-section and the velocity of liquid at any point in a tube of flow. What conclusion do you draw from it?

36. State Stoke's law for viscous drag experienced by the spherical body falling through a viscous liquid. Why does a spherical body achieve terminal speed? On what factors does the terminal speed depends.

OR

Define surface tension and surface energy. Obtain a relation between them.

37. (a) Describe elastic hysteresis. Mention its two applications.

(b) What is elastic after-effect?

OR

A wire of cross-sectional area A is stretched horizontally between two clamps located at a distance $2l$ metres from each other. A weight W kg is suspended from the midpoint of the wire. If the vertical distance through which the mid-point of the wire moves down be $x < l$, then find (i) the strain produced in the wire, (ii) the stress on the wire. (iii) If Y is the Young's modulus of wire, then find the value of x.

SOLUTIONS

1. (d) **2.** (a) **3.** (d) **4.** (b) **5.** (d)
6. (a) **7.** (b) **8.** (c) **9.** (c) **10.** (c)

11. When a wire is bent back and forth, its deformations are beyond the elastic limit. The work done against interatomic forces is no longer stored totally in the form of potential energy. The crystalline structure of the wire gets affected and work done is converted into heat energy.

12. Like liquids, amorphous solids have disordered arrangement of atoms or molecules. The molecules of liquid are free to move but the molecules of an amorphous solids are almost fixed at their positions *i.e.*, amorphous solids are rigid due to their high viscosity. That is why, we can say amorphous solids are super-cooled liquids of high viscosity.

13. When spring balances have been used for a long time, they develop elastic fatigue in them. The springs of such balances will take more time to recover their original configurations. Thus, the readings shown by such balances will be wrong.

14. The weight of air displaced by the bag is same as the weight of air inside it. The increase in weight due to the filled air gets cancelled by the upthrust of air. So weight remains same when air is filled in the bag.

15. When a fast train crosses the platform, the air dragged along with the train also moves with a high velocity. In accordance with Bernoulli's equation, the pressure in the region of high velocity decreases. If a person stands near the edge of the platform he may be pushed towards the train due to high pressure outside.

16. The atmospheric pressure is low at high altitudes. Due to greater pressure difference in blood pressure and the atmospheric pressure at high altitude, it is difficult to stop bleeding from a cut in the body.

17. Since $Y_{steel} = 200 \times 10^9$ N m^{-2}
and $Y_{copper} = 110 \times 10^9$ N m^{-2}.

Young's modulus of steel is more than that of copper.

$\because$ $F = YA\,\Delta l/l$; $F \propto Y$ for the given value of Δl, A and l.

Therefore to stretch them equally more force will be required on the steel spring than that on copper spring.

Hence work done, $W = \frac{1}{2} F\,\Delta l$,

or $W \propto F$ ($\because$ Δl is constant)

Thus more work will have to be done on steel wire than on copper wire.

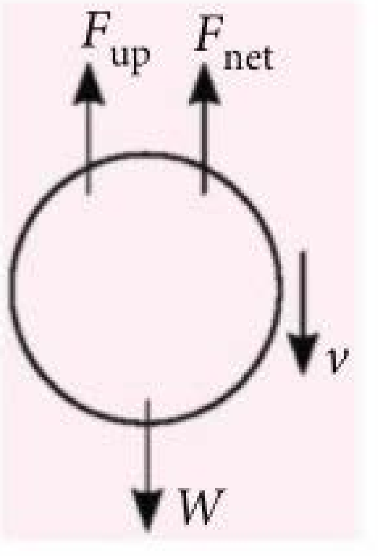
18. When water enters into a narrow pipe, the area of cross-section (A) decreases and consequently velocity (v) increases as Av = constant.

19. Under constant stress, magnitude of bulk modulus of elasticity of material is inversely proportional to volumetric strain.

$$B \propto \frac{1}{\Delta V / V}$$

Since, gases has higher compressibility than liquid and solids have the least, therefore $B_s > B_l > B_g$.

20. Let the liquid has viscosity η



$F_{up} = \sigma V g$
$W = \rho V g$
$F = 6\pi\eta r v$

Initially when the ball is released ($\eta \approx 0$)

$F_{net} = W - F_{up}$

$ma = mg - F_{up}$, where $m = \rho V$

$$a = g - \frac{F_{up}}{m} = g - \frac{\sigma}{\rho} g$$

21. Consider two rods of steel and rubber, each having length l and area of cross-section A. If they are subjected to the same deforming force F, then the extension Δl_s produced in the steel rod will be less than the extension Δl_r in the rubber rod, *i.e.*, $\Delta l_s < \Delta l_r$. Now

$$Y_s = \frac{F}{A}.\frac{l}{\Delta l_s} \text{ and } Y_r = \frac{F}{A}.\frac{l}{\Delta l_r}$$

$$\therefore \frac{Y_s}{Y_r} = \frac{\Delta l_r}{\Delta l_s}$$

As $\Delta l_s < \Delta l_r$, So $Y_s > Y_r$

i.e., Young's modulus for steel is greater than that of rubber. Hence steel is more elastic than rubber.

22. Pressure at orifice,

$P = 500 \text{ gf cm}^{-2}$

$$= \frac{500}{1000} \times 9.8 \times (100)^2 \text{ N m}^{-2} = 500 \times 98 \text{ N m}^{-2}$$

Let h be the depth of orifice below the surface.

As, $P = h \rho g$

$$\therefore h = \frac{P}{\rho g} = \frac{500 \times 98}{10^3 \times 9.8} = 5 \text{ m}$$

The velocity of efflux,

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 5} = 9.893 \text{ m s}^{-1}$$

23. (a) Young's modulus,

$$Y = \frac{Mgl}{\pi r^2.\Delta l} = \frac{Mgl}{\pi\left(\frac{D}{2}\right)^2 \Delta l} = \frac{4Mgl}{\pi D^2 \Delta l}$$

where D is the diameter of the wire.

$$\Delta l = \frac{4Mgl}{\pi D^2 Y} \quad \text{...(i)}$$

From (i), $\Delta l \propto \frac{1}{D^2}$

Clearly, if the diameter is doubled, the elongation will become one-fourth.

(b) Also load, $Mg = \frac{\pi D^2 \Delta l.Y}{4l}$ *i.e.*, $Mg \propto D^2$

Clearly, if the diameter is doubled, the wire can support 4 times the original load.

OR

(a) When two wires of same size are suspended in parallel, a force F equal to the breaking force will act on each wire if a breaking force of $2F$ is applied on the parallel combination.

(b) $F = \frac{YA\Delta l}{l} = \frac{Y.\pi r^2 \Delta l}{l}$ *i.e.*, $F \propto r^2$

Thus for a single wire of double the thickness, the breaking force will be $4F$.

24. Equating the rate of flow, $v_1 A_1 = v_2 A_2$

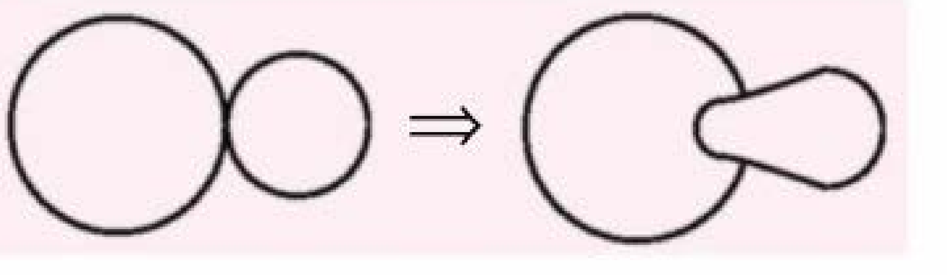
But $v_1 = \sqrt{2gy}, A_1 = L^2, v_2 = \sqrt{2g \times 4y},\ A_2 = \pi R^2$

$$\therefore \sqrt{2gy} \times L^2 = \sqrt{2g \times 4y} \times \pi R^2$$

(using Torricelli's law)

or $L^2 = 2\pi R^2$ or $R = \frac{L}{\sqrt{2\pi}}$

25. When seen from inside the smaller bubble, the shape of the common boundary will appear concave, as shown in figure.



Reasons: (i) For a curved liquid film, the pressure is greater on its concave side.

(ii) Pressure inside the smaller bubble is more than that inside the larger bubble, because $P \propto 1/r$.

OR

Surface energy = surface tension × surface area,

$E = S \times 2A$ ($\because$ Soap film has two surfaces)

As $A' = \left[A - \frac{50A}{100}\right] = \frac{A}{2}$

$$\therefore E' = S \times 2\left(\frac{A}{2}\right) = SA$$

Percentage decrease in surface energy

$$= \frac{E - E'}{E} \times 100\% = \frac{2SA - SA}{2SA} \times 100\%$$

$$= \frac{1}{2} \times 100\% = 50\%$$

26. $\Delta l_P = \frac{3mg}{A} \times \frac{L}{Y}$

$$\Delta l_Q = \frac{2\,mg}{A}.\frac{2L}{Y/2} = \frac{8\,mg}{A}\frac{L}{Y} \quad \therefore \frac{\Delta l_P}{\Delta l_Q} = \frac{3}{8}$$

27. Bulk modulus, $B = \frac{P}{dV/V}$ or $dV = \frac{PV}{B}$;

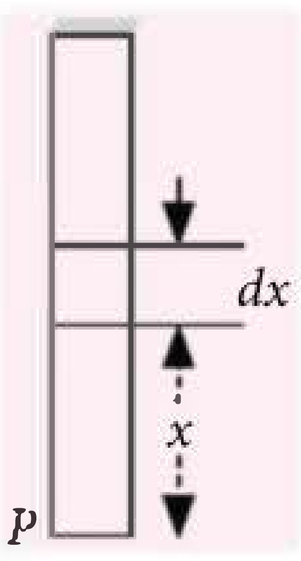
When pressure is increased, the volume will decrease.

Therefore, $\rho = \frac{M}{V}$ and $\rho' = \frac{M}{V - dV} = \frac{M}{V - (PV/B)}$

or $\rho' = \frac{M}{V(1 - P/B)} = \frac{\rho}{(1 - P/B)}$

or $\frac{\rho'}{\rho} = \frac{1}{(1 - P/B)} = (1 - P/B)^{-1} = \left(1 + \frac{P}{B}\right)$

28. Since the tension is non-uniform along the wire, we have variable stress in wire. Let us consider a small element of length dx at a distance x from bottom end P as shown figure.



The tension in the element of the wire

$$= \frac{mg}{l}x$$

If dl is increase in length of the element,

then longitudinal strain $= \frac{dl}{dx}$;

Normal stress $= \frac{mgx}{l} \times \frac{1}{\pi r^2}$

$$\therefore \quad Y = \frac{mgx}{l\pi r^2} \times \frac{dx}{dl} \quad \text{or} \quad dl = \frac{mgx\,dx}{Y\pi r^2 l}$$

Total change in length of wire is

$$\Delta l = \int_0^l \frac{mgx\,dx}{Y\pi r^2 l} = \frac{mg}{Y\pi r^2 l} \times \frac{l^2}{2} = \frac{mgl}{2Y\pi r^2}$$

29. Let r_1 and r_2 be the radii of the two wires.

(i) Stress $= \frac{F}{A} = \frac{F}{\pi r^2}$. For same load F, $\frac{(\text{stress})_1}{(\text{stress})_2} = \frac{r_2^2}{r_1^2}$

(ii) Strain, $\frac{\Delta l}{l} = \frac{F}{AY} = \frac{F}{\pi r^2 Y}$

For the two wires F and Y are same, so $\frac{(\text{strain})_1}{(\text{strain})_2} = \frac{r_2^2}{r_1^2}$

(iii) Extension, $\Delta l = \frac{F}{A} \cdot \frac{l}{Y} = \frac{F}{\pi r^2} \cdot \frac{l}{Y}$

For the two wires, F, l and Y are same, so $\frac{(\Delta l)_1}{(\Delta l)_2} = \frac{r_2^2}{r_1^2}$

Hence stress, strain and extension are all different for the two wires.

OR

(a) The resolved part of F along the normal is the tensile force on this plane and the resolved part parallel to the plane is the shearing force on the plane.

Tensile stress $= \frac{\text{Force}}{\text{Area}} = \frac{F\cos\theta}{A\sec\theta} = \frac{F}{A}\cos^2\theta$

[$\because$ Area of plane section $= A\sec\theta$]

(b) Shearing stress

$$= \frac{\text{Force}}{\text{Area}} = \frac{F\sin\theta}{A\sec\theta} = \frac{F}{A}\sin\theta\cos\theta = \frac{F}{2A}\sin 2\theta.$$

(c) Tensile stress will be maximum when $\cos^2\theta$ is maximum, *i.e.*, $\cos\theta = 1$ or $\theta = 0°$.

30. Let m, M be the mass of stone and ice piece respectively. As ice piece with stone of mass $(m + M)$ floats in water, so mass of water displaced is $(m + M)$. If ρ_w is the density of water, then volume of water displaced is

$$V = \frac{M+m}{\rho_w} \quad \text{...(i)}$$

When the ice melts completely, there will be extra water of mass M and volume M/ρ_w in the beaker. Now the stone will sink and will displace water equal to its volume $(= m/\rho_s)$, where ρ_s is the density of stone. Thus the total volume of extra water obtained by melting of ice and displaced by sinking stone is

$$V' = \frac{M}{\rho_w} + \frac{m}{\rho_s} \quad \text{...(ii)}$$

As $\rho_s > \rho_w$ so $\frac{1}{\rho_s} < \frac{1}{\rho_w}$ and $\frac{m}{\rho_s} < \frac{m}{\rho_w}$

From (i) and (ii) we note that ;

$V' < V$. It means the level of water in the beaker will come down.

31. Here, $P_1 = 4 \times 10^4$ N m^{-2},

$v_1 = 2$ m s^{-1}, $a_1 = 0.02$ m^2

$a_2 = 0.01$ m^2 ; $P_2 = ?$

As, $a_1 v_1 = a_2 v_2$

so, $v_2 = \frac{a_1 v_1}{a_2} = \frac{0.02 \times 2}{0.01} = 4$ m s^{-1}

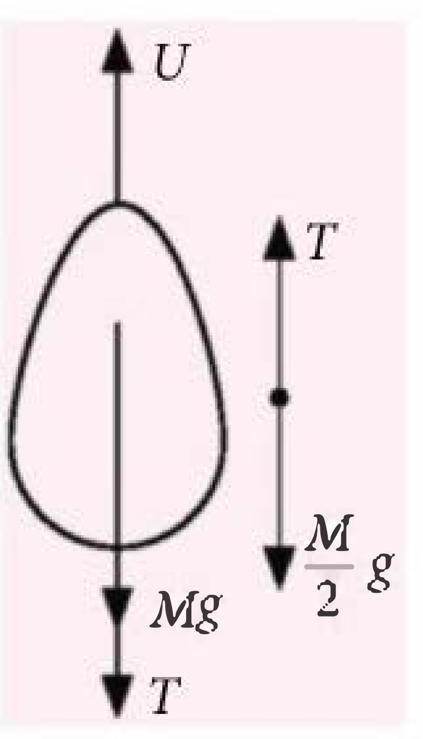
Now, $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$$P_2 = P_1 + \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2 = P_1 - \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$P_2 = 4 \times 10^4 - \frac{1}{2} \times 10^3 [4^2 - 2^2]$$

$= 4 \times 10^4 - 0.6 \times 10^4 = 3.4 \times 10^4$ Pa

32. The situation is as shown in the figure.



For the balloon to be held stationary in air, the forces acting on it should be balanced.

Upthrust = Weight of balloon + Tension in the string

$$U = Mg + T \quad \text{...(i)}$$

For the small block of mass $M/2$ floating stationary in air,

$$T = \frac{M}{2}g \quad \text{...(ii)}$$

From (i) and (ii) we get, $U = Mg + \frac{M}{2}g = \frac{3}{2}Mg$

When the string is cut, $T = 0$. The small block will begin to fall freely. The balloon will rise up with an acceleration a such that

$$U - Mg = Ma \quad \text{or} \quad \frac{3}{2}Mg - Mg = Ma$$

or $a = \frac{g}{2}$, in the upward direction.

OR

Let h be the height of container at which its blood exerts pressure equal to gauge pressure in vein. Then

$$h\rho g = P_g$$

or $h = \frac{P_g}{\rho g} = \frac{2000}{1.06 \times 10^3 \times 9.8} = 0.1925 \text{ m}$

The blood will just enter the vein if the blood container is kept at height slightly greater than 0.1925 m *i.e.*, at 0.2 m.

33. (a) Wire of material A with larger plastic region is more ductile.

(b) Young's modulus is $\frac{\text{Stress}}{\text{Strain}}$

$\therefore \quad Y_A > Y_B$

(c) For a given strain, larger stress required for A than that for B.

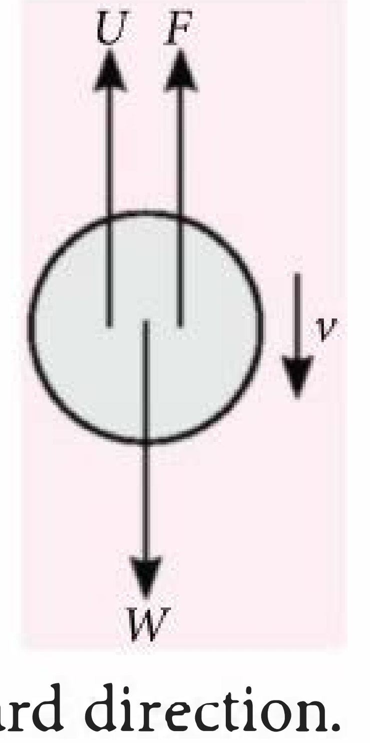
$\therefore \quad A$ is stronger than B.

(d) Material with smaller plastic region is more brittle, therefore B is more brittle than A.

34. Terminal velocity : It is the maximum constant velocity acquired by the drop while falling through a viscous medium.

Expression for terminal velocity : Consider a spherical body of radius r falling through a viscous liquid of density σ and coefficient of viscosity η. Let ρ be the density of the body.

As the body falls, the various forces acting on the body are as shown in figure. These are



(i) Weight of the body acting vertically downwards.

$$W = mg = \frac{4}{3}\pi r^3 \rho g$$

(ii) Upward thrust equal to the weight of the liquid displaced.

$$U = \frac{4}{3}\pi r^3 \sigma g$$

(iii) Force of viscosity F acting in the upward direction. According to Stokes' law, $F = 6\pi\eta r v$

Clearly, the force of viscosity increases as the velocity of the body increases. A stage is reached, when the weight of the body becomes just equal to the sum of the upthrust and the viscous force. Then the body begins to fall with a constant maximum velocity, called terminal velocity.

When the body attains terminal velocity v, $U + F = W$

$$\frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r v = \frac{4}{3}\pi r^3 \rho g$$

or $6\pi\eta r v = \frac{4}{3}\pi r^3 (\rho - \sigma) g$ or $v = \frac{2}{9} \cdot \frac{r^2(\rho - \sigma) g}{\eta}$

Thus, the value of terminal velocity v depends upon r, ρ, σ and η.

35. *Refer to answer 59 page no. 284 (CBSE Champion Class 11).*

OR

Refer to answer 34 page no. 280 (CBSE Champion Class 11).

36. *Refer to answer 85 page no. 286 (CBSE Champion Class 11).*

OR

Refer to answer 145 page no. 292 (CBSE Champion Class 11).

37. *Refer to answer 27 page no. 253 (CBSE Champion Class 11).*

OR

Refer to answer 102 page no. 263 (CBSE Champion Class 11).

MONTHLY TEST DRIVE

XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Laws of Motion | Work, Energy and Power

Total Marks : 120 **Time Taken : 60 Min.**

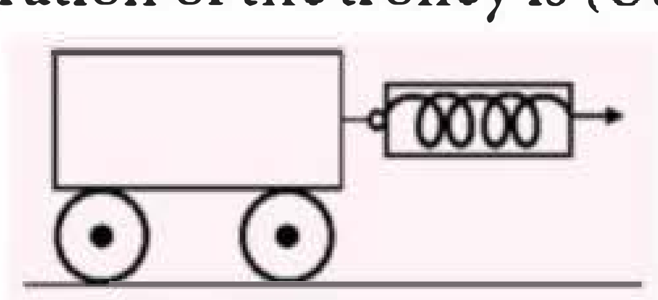
NEET / AIIMS

Only One Option Correct Type

1. A particle of mass m is tied to a light string of length l and rotated along a vertical circular path. What should be the minimum speed at the highest point of its path so that the string does not become slack at any position?

(a) $\sqrt{2gl}$ (b) $\sqrt{gl}$ (c) zero (d) $\sqrt{gl/2}$

2. A massless spring balance is attached to 2 kg trolley and is used to pull the trolley along a flat smooth surface as shown in the figure. The reading on the spring balance remains at 10 kg during the motion. The acceleration of the trolley is (Use $g = 9.8$ m s^{-2})



(a) 4.9 m s^{-2} (b) 9.8 m s^{-2}
(c) 49 m s^{-2} (d) 98 m s^{-2}

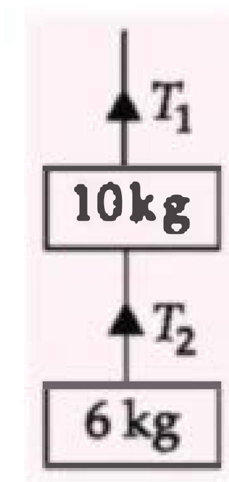
3. In a simple pendulum, the breaking strength of the string is double the weight of the bob. The bob is released from rest when the string is horizontal. The string breaks when it makes an angle θ with the vertical. Then,

(a) $\theta = \cos^{-1}(1/3)$ (b) $\theta = 60°$
(c) $\theta = \cos^{-1}(2/3)$ (d) $\theta = 75°$

4. A horizontal force of 12 N pushes a 0.5 kg book against a vertical wall. The book is initially at rest. If the coefficients of friction are $\mu_k = 0.6$ and $\mu_s = 0.8$, which of the following statements is true?

(a) The magnitude of the frictional force is 5 N.
(b) The magnitude of the frictional force is 7.2 N.
(c) The normal force is 5 N.
(d) The book will start moving and will accelerate.

5. A body of mass 6 kg is hanging from another of mass 10 kg as shown in figure. This combination is being pulled up by a string with an acceleration of 2 m s^{-2}. The tension T_1 is ($g = 10$ m s^{-2})

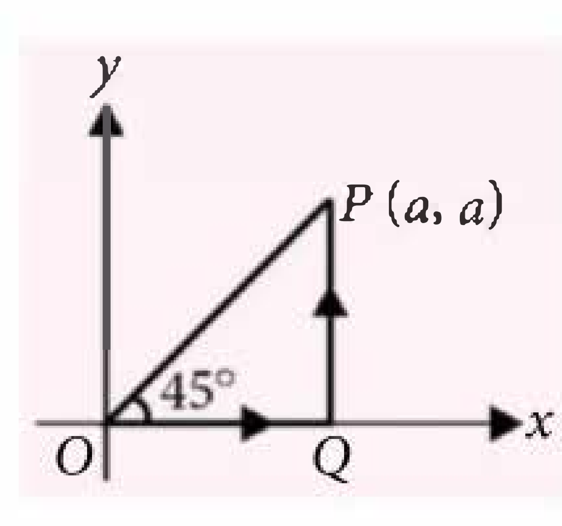


(a) 240 N (b) 150 N
(c) 220 N (d) 192 N

6. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time as $a_c = k^2rt^2$, where k is a constant. The power delivered to the particle by the forces acting on it is

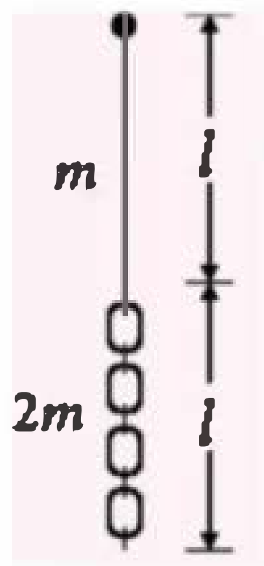
(a) $2\pi mk^2r^2t$ (b) mk^2r^2t
(c) $\frac{1}{3}mk^4r^2t^5$ (d) 0

7. A particle is moved from (0, 0) to (a, a) under a force $\vec{F} = (3\hat{i} + 4\hat{j})$ from two paths. Path 1 is OP and path 2 is OQP. Let W_1 and W_2 be the work done by this force in these two paths. Then,



(a) $W_1 = W_2$ (b) $W_1 = 2W_2$
(c) $W_2 = 2W_1$ (d) $W_2 = 4W_1$

8. A rope of length l and mass m is connected to a chain of length l and mass 2 m, and hung vertically as shown in figure. What is the change in gravitational potential energy if the system is inverted and hung from same point?



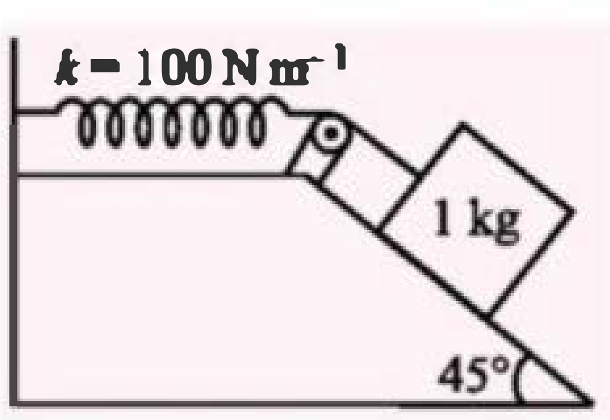
(a) mgl (b) 1.5 mgl (c) 0.5 mgl (d) 2 mgl

9. In a ballistics demonstration a police officer fires a bullet of mass 50.0 g with speed 200 m s^{-1} on soft plywood of thickness 2.00 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet?

(a) $2\sqrt{10}$ m s^{-1} (b) $20\sqrt{10}$ m s^{-1}
(c) $10\sqrt{2}$ m s^{-1} (d) $10\sqrt{20}$ m s^{-1}

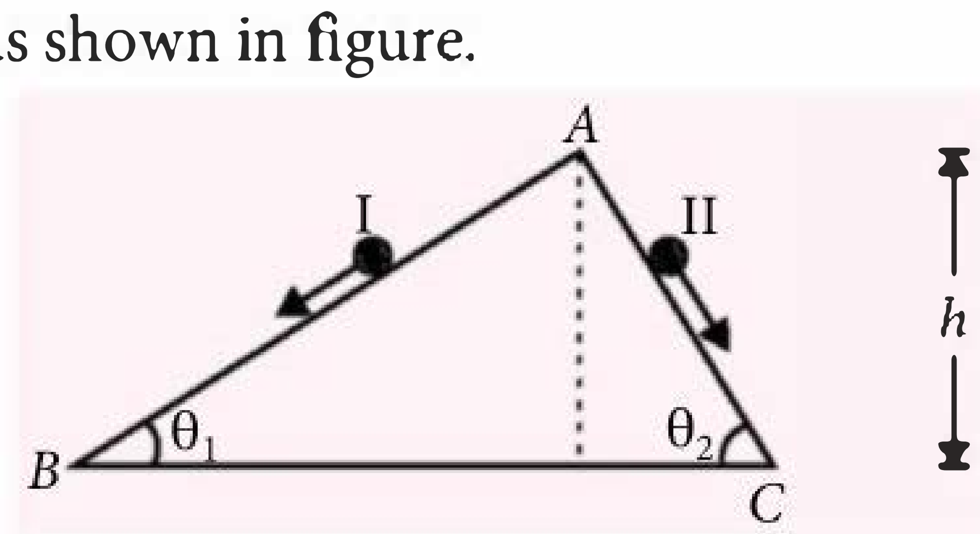
10. A large force is acting on a body for a short time. The impulse imparted is equal to the change in
(a) acceleration (b) momentum
(c) energy (d) velocity.

11. A 1 kg block situated on a rough incline is connected to a spring of negligible mass having spring constant 100 N m^{-1} as shown in the figure. The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. The coefficient of friction between the block and the incline is (Take g = 10 m s^{-2} and assume that the pulley is frictionless)



(a) 0.2 (b) 0.3 (c) 0.5 (d) 0.6

12. Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track as shown in figure.



Which of the following statement is correct?
(a) Both the stones reach the bottom at the same time but not with the same speed.
(b) Both the stones reach the bottom with the same speed and stone I reaches the bottom earlier than stone II.
(c) Both the stones reach the bottom with the same speed and stone II reaches the bottom earlier than stone I.
(d) Both the stones reach the bottom at different times and with different speeds.

Assertion and Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as:
(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

13. **Assertion :** Pseudo force is an imaginary force which is recognised only by a non-inertial observer to explain the physical situation according to Newton's laws.
Reason : Pseudo force has no physical origin, *i.e.*, it is not caused by one of the basic interactions in nature. It does not exist in the action-reaction pair.

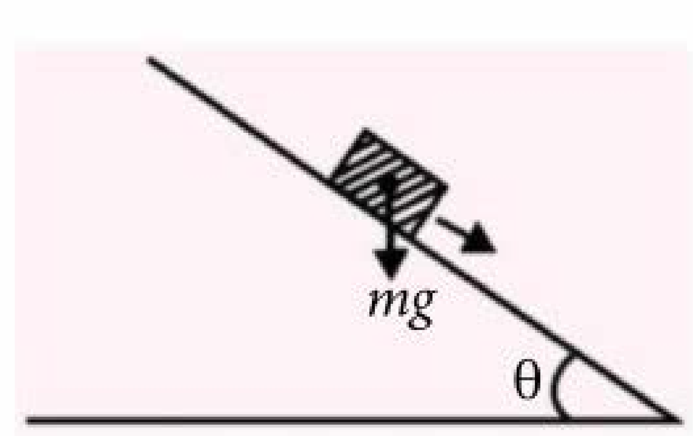
14. **Assertion :** A quick collision between two bodies is more violent than a slow collision, even when the initial and the final velocities are identical.
Reason : The rate of change of momentum determines the force is greater in a quick collision between two bodies.

15. **Assertion :** All central forces which follow the inverse square law are conservative forces.
Reason : Work done by the force or against the force does not depend on path, then force is called conservative force.

JEE MAIN / ADVANCED

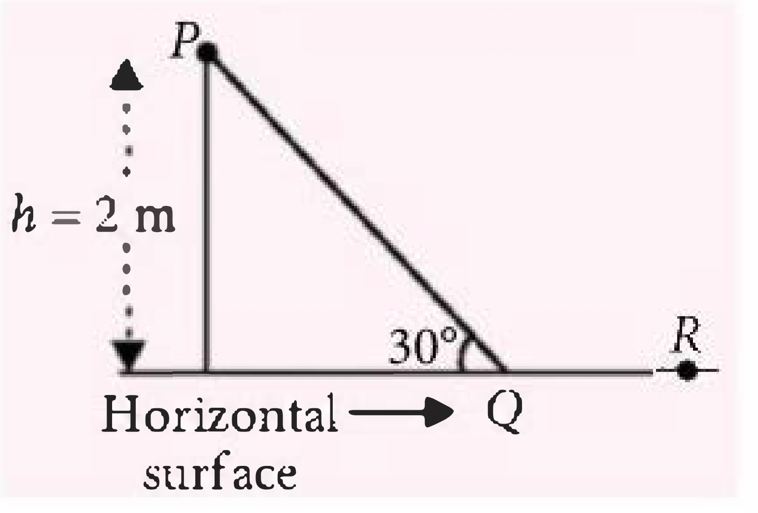
Only One Option Correct Type

16. A plank with a box on it at one end is gradually raised about the other end. As the angle of inclination with the horizontal reaches 30°, the box starts to slip and slides 4.0 m down the plank in 4.0 s. The coefficients of static and kinetic friction between the box and the plank will be, respectively (Take g = 10 m s^{-2})



(a) 0.5 and 0.6 (b) 0.4 and 0.3
(c) 0.6 and 0.6 (d) 0.6 and 0.5

17. A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction between the particle and the rough track is μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The values of the coefficient of friction μ and the distance x ($= QR$) are, respectively close to



(a) 0.2 and 6.5 m (b) 0.2 and 3.5 m
(c) 0.29 and 3.5 m (d) 0.29 and 6.5 m

18. A railway track is banked for a speed v, by making the height of the outer rail h higher than that of the inner rail. The distance between the rails is d. The radius of curvature of the track is r. Then,

(a) $\frac{h}{d}=\frac{v^2}{rg}$ (b) $\tan\left(\sin^{-1}\frac{h}{d}\right)=\frac{v^2}{rg}$

(c) $\tan^{-1}\left(\frac{h}{d}\right)=\frac{v^2}{rg}$ (d) $\frac{h}{r}=\frac{v^2}{dg}$

19. A stiff spring has a force law given by $F = -kx^3$. The work required to stretch the spring from the relaxed state $x = 0$ to the stretched length $x = l$ is W_0. In terms of W_0, how much work is required to extend the spring from the stretched length l to the length $2l$?

(a) W_0 (b) $4\,W_0$
(c) $10\,W_0$ (d) $15\,W_0$

More than One Options Correct Type

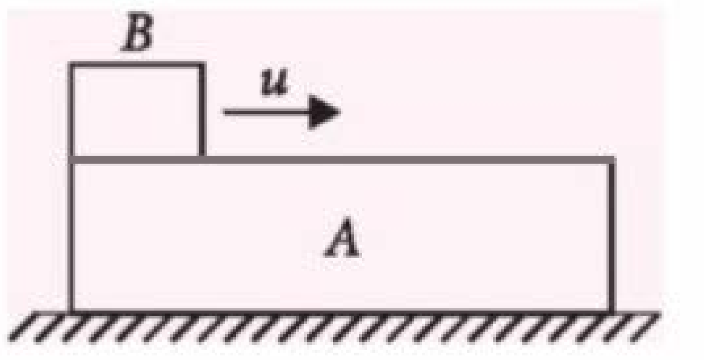
20. A force $\vec{F}$ (larger than the limiting friction force) is applied to the left of an object moving to the right on a rough horizontal surface. Then,

(a) initially the object would slow down
(b) for some time $\vec{F}$ and friction force will act in the same direction and for remaining time they act in opposite directions
(c) the object comes to rest for a moment and after that its motion is accelerating in the direction of $\vec{F}$
(d) the object slows down and finally comes to rest.

21. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 m s^{-1}. Which of the following statement(s) is (are) correct for the system of these two masses?

(a) Total momentum of the system is 3 kg m s^{-1}.
(b) Momentum of 5 kg mass after collision is 4 kg m s^{-1}.
(c) Kinetic energy of the centre of mass is 0.75 J.
(d) Total kinetic energy of the system is 4 J.

22. A long block A is at rest on a smooth horizontal surface. A small block B, whose mass is half of A, is placed on A at one end and projected along A with some velocity u. The coefficient of friction between the blocks is μ. Then,



(a) the blocks will reach a final common velocity $u/3$
(b) the work done against friction is two-third of the initial kinetic energy of B
(c) before the blocks reach a common velocity, the acceleration of A relative to B is $\frac{2}{3}\mu g$
(d) before the blocks reach a common velocity the acceleration of A relative to B is $\frac{3}{2}\mu g$.

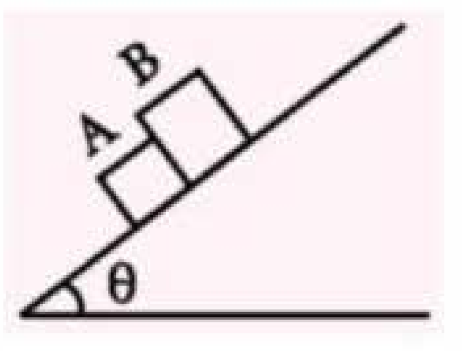
23. A stone of weight W is thrown vertically upward into the air with an initial speed v_0. Suppose that the air drag force f dissipates an amount fy of mechanical energy as the stone travels a distance y. Then,

(a) the maximum height reached by the stone is $\frac{v_0^2}{2g(1+f/W)}$
(b) the maximum height reached by stone is $\frac{v_0^2}{2g(4+f/W)}$
(c) the speed of the stone upon impact with the ground is $v_0\left(\frac{W-f}{W+f}\right)^{1/2}$
(d) the speed of the stone upon impact with the ground is $v_0\left(\frac{W+f}{W-f}\right)^{1/2}$.

24. Two balls, having linear momenta $\vec{p}_1 = p\hat{i}$ and $\vec{p}_2 = -p\hat{i}$, undergo a collision in free space. There is no external force acting on the balls. Let $\vec{p}_1'$ and $\vec{p}_2'$ be their final momenta. The following options are not allowed for any non-zero value of p, a_1, a_2, b_1, b_2, c_1 and c_2.

(a) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}_2' = a_2\hat{i} + b_2\hat{j}$
(b) $\vec{p}_1' = c_1\hat{k}$, $\vec{p}_2' = c_2\hat{k}$
(c) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}_2' = a_2\hat{i} + b_2\hat{j} - c_1\hat{k}$
(d) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j}$, $\vec{p}_2' = a_2\hat{i} + b_1\hat{j}$

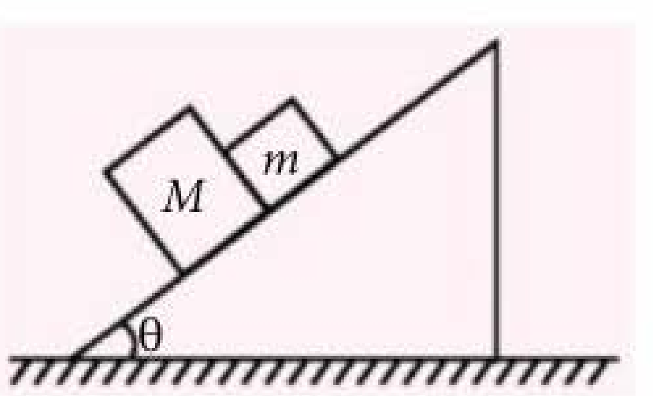
25. The two blocks A and B of equal mass are initially in contact when released from rest on the inclined plane. The coefficients of friction between the inclined plane and A and B are μ_1 and μ_2 respectively. Then,



(a) if $\mu_1 > \mu_2$, the blocks will always remain in contact
(b) if $\mu_1 < \mu_2$, the blocks will slide down with different accelerations
(c) if $\mu_1 > \mu_2$, the blocks will have a common acceleration $\frac{1}{2}(\mu_1 + \mu_2) g \sin\theta$
(d) if $\mu_1 < \mu_2$, the blocks will have a common acceleration $\frac{\mu_1 \mu_2 g}{\mu_1 + \mu_2} \sin\theta$.

Numerical Value Type

26. Figure shows that two blocks in contact are sliding down an inclined surface of inclination $\theta = 30°$. The friction coefficient between the block of mass $m = 2$ kg and the incline is $\mu_1 = 0.20$ and that between the block of mass $M = 4$ kg and the incline is $\mu_2 = 0.30$. Find the acceleration (in m s^{-2}) of 2 kg block. (Take $g = 10$ m s^{-2})

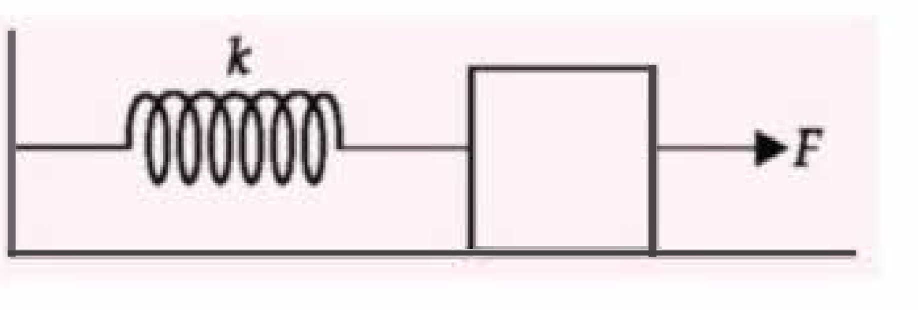


27. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$ where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U_{(x=\infty)} - U_{\text{equilibrium}}] = \frac{b^2}{pa}$, then find the value of p.

28. A mass of 1 kg is suspended by means of a thread. The system is (i) lifted up with an acceleration of 4.9 m s^{-2} (ii) lowered with an acceleration of 4.9 m s^{-2}. The ratio of tension in the first and second case is $x : 1$. Find x.

Comprehension Type

The spring shown in figure is unstretched when a man starts pulling the block. The mass of the block is M. If the man stretch by a constant force F and releases.



29. The amplitude and time period of the motion of the block is

(a) $\frac{F}{k}, 2\pi\sqrt{\frac{M}{k}}$
(b) $\frac{F}{2k}, 2\pi\sqrt{\frac{M}{2k}}$
(c) $\frac{2F}{k}, 2\pi\sqrt{\frac{M}{4k}}$
(d) none of these

30. The energy stored in spring when the block passes through the equilibrium position is

(a) $\frac{2F^2}{k}$ (b) $\frac{F^2}{k}$ (c) $\frac{F^2}{4k}$ (d) $\frac{F^2}{2k}$

❖❖

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CHECK YOUR PERFORMANCE

No. of questions attempted ......
No. of questions correct ......
Marks scored in percentage ......

If your score is

> 80%	Your preparation is going good, keep it up to get high score.
60-80%	Need more practice, try hard to score more next time.
<60%	Stress more on concepts and revise thoroughly.

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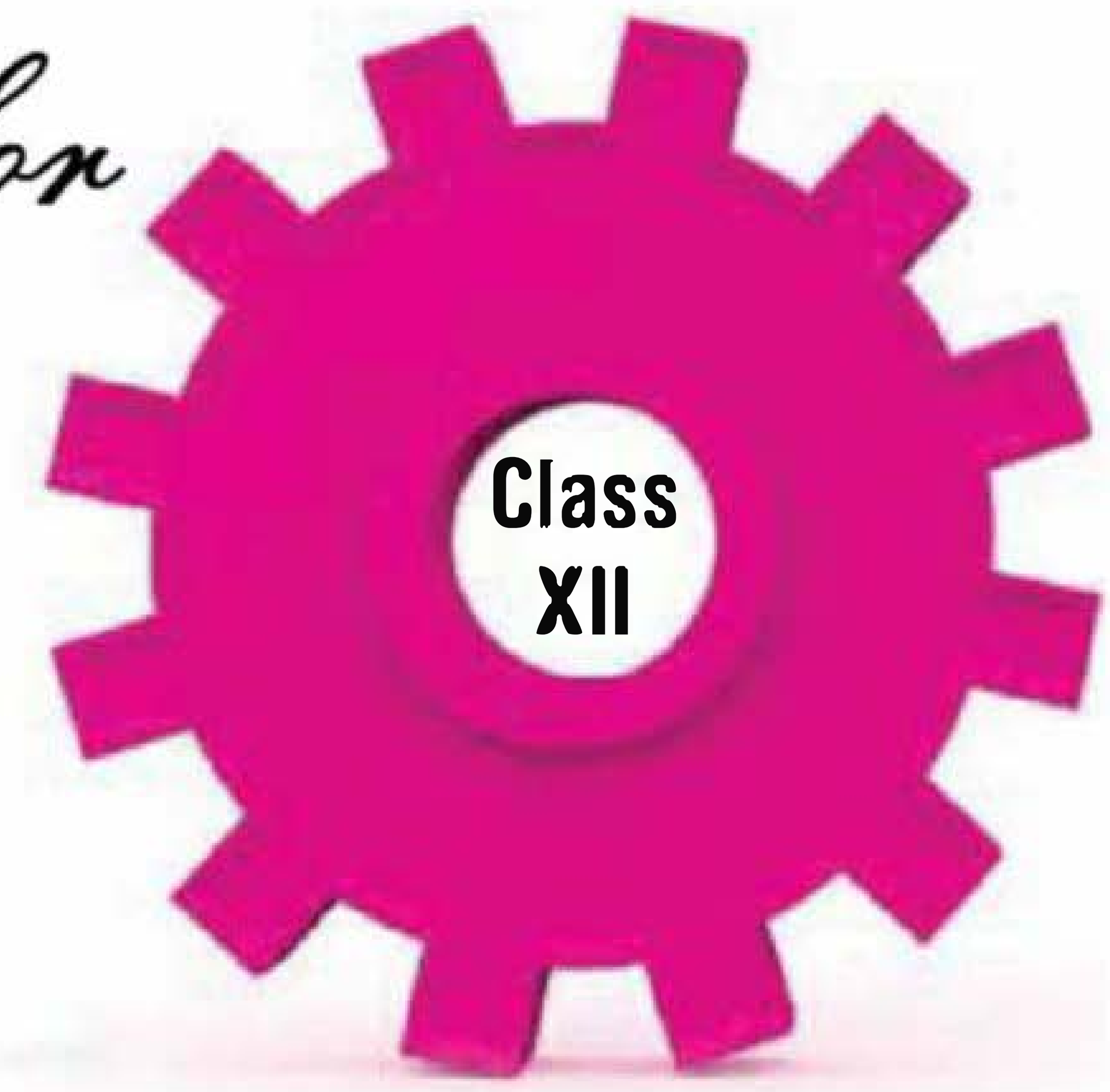

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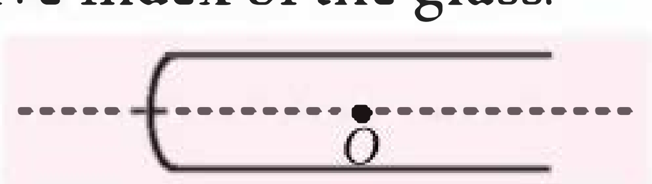
Practicing these MCQs help to strengthen your concepts and give you extra edge in your JEE preparation

1. An object O is in a glass medium. The refraction take place at the concave boundary between glass and air. The object distance is greater than 5/2 R (R is radius of curvature) for image to be real. Find the refractive index of the glass.



(a) 3/2 (b) 1.75 (c) 5/3 (d) 2.0

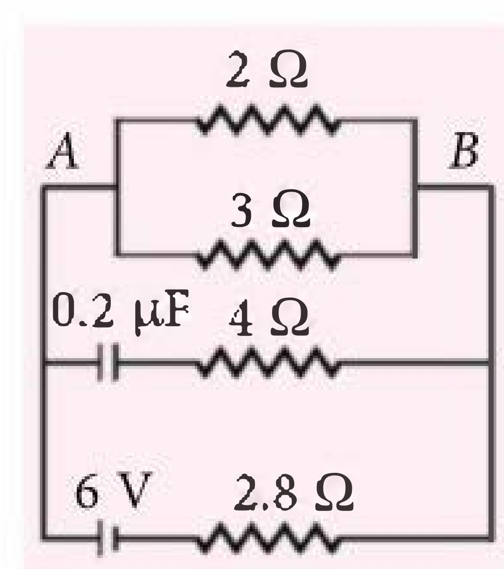
2. Radiation from the Sun reaches Earth at the rate of 1350 J s^{-1} m^{-2}. The magnitudes of electric and magnetic field are
(a) 1.01×10^6 V m^{-1}, 3.37×10^{-3} T
(b) 3.37×10^3 V m^{-1}, 1.01×10^{-6} T
(c) 1.01×10^3 V m^{-1}, 3.37×10^{-6} T
(d) 3.37×10^6 V m^{-1}, 1.01×10^{-3} T

3. The first member of the Paschen series in hydrogen spectrum is of wavelength 18800 Å. The shortest wavelength of Paschen series is
(a) 1215 Å (b) 6560 Å (c) 8182 Å (d) 12850 Å

4. A small electric dipole is placed at origin with its dipole moment directed along positive X-axis. The direction of electric field at point $(2, 2\sqrt{2}, 0)$ is
(a) along negative Z-axis
(b) along Z-axis
(c) along negative Y-axis
(d) along positive Y-axis.

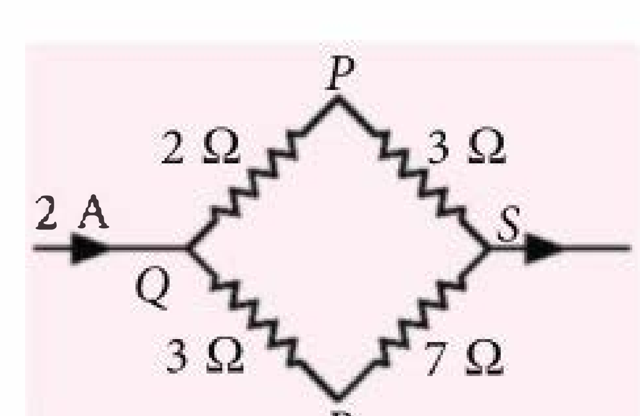
5. In a double slit arrangement, fringes are produced using light of wavelength 4800 Å. One slit is covered by a thin plate of glass of refractive index 1.4 and the other with another glass plate of same thickness but of refractive index 1.7. By doing so, the central bright fringe shifts to original fifth bright fringe from the centre. Thickness of glass plate is
(a) 8 μm (b) 6 μm (c) 4 μm (d) 10 μm

6. Calculate the steady state current in the 2 Ω resistor shown in the given figure. The internal resistance of the battery is negligible and the capacitance of the capacitor is 0.2 μF.



(a) 0.9 A (b) 2.5 A (c) 2.0 A (d) 3 A

7. A current of 2 A flows in the system of conductors as shown in figure. The potential difference $V_P - V_R$ will be



(a) −2 V (b) −0.67 V (c) +6 V (d) +6.7 V

8. Two straight parallel conductors are at a distance of 50 cm perpendicular to the plane of paper. They carry current of 20 A and 30 A respectively in opposite directions. A point P is separated from first conductor by a distance of 40 cm and from the second conductor by 30 cm. The magnetic field induction at point P is

(a) 2×10^5 T (b) $\sqrt{5} \times 10^{-5}$ T
(c) 3×10^5 T (d) $\sqrt{8} \times 10^{-5}$ T

9. The half-life of a sample of a radioactive substance is 1 hour. If 8×10^{10} atoms are present at $t = 0$, then the number of atoms decayed in the duration $t = 2$ hour to $t = 4$ hour will be

(a) 2×10^{10} (b) 1.5×10^{10}
(c) zero (d) infinity

10. An electron is fired directly towards the centre of a large metal plate that has excess negative charge with surface charge density 2.0×10^{-6} C m^{-2}. If the initial kinetic energy of electron is 100 eV, and if it is to stop due to repulsion just as it reaches the plate, how far from the plate must it be fired?

(a) 2.5×10^{-6} m (b) 3×10^{-7} m
(c) 1.7×10^{-3} m (d) 4.43×10^{-4} m

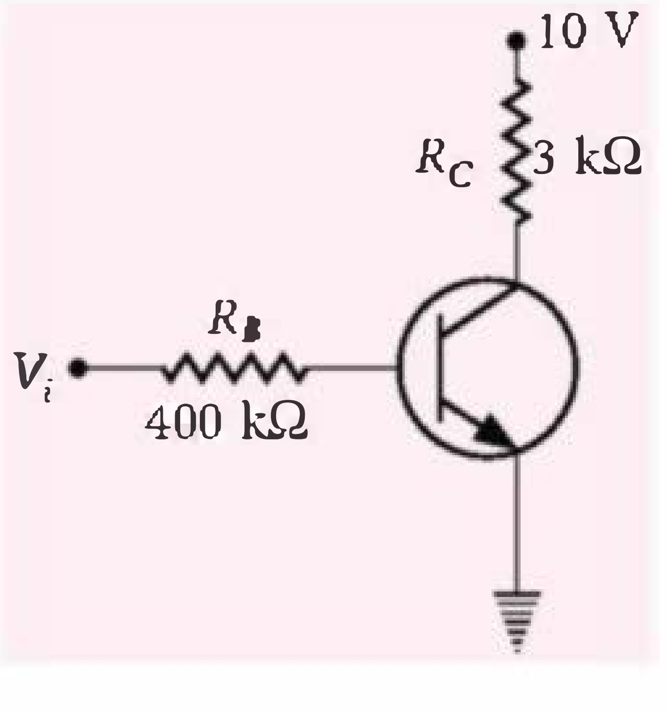
11. Over a solenoid of 50 cm length and 2 cm radius and having 500 turns, is wound another wire of 50 turns, coaxially. Calculate the induced emf in the second coil when the current in the primary changes from 0 to 5 A in 0.02 s.

(a) 19.72 mV (b) 9.7 mV
(c) 190 mV (d) 1.10 mV

12. Alternating emf, $\varepsilon = 220 \sin 100\pi t$ is applied to a circuit containing an inductance of $1/\pi$ H. What will will be the reading of an ac ammeter if connected in the circuit?

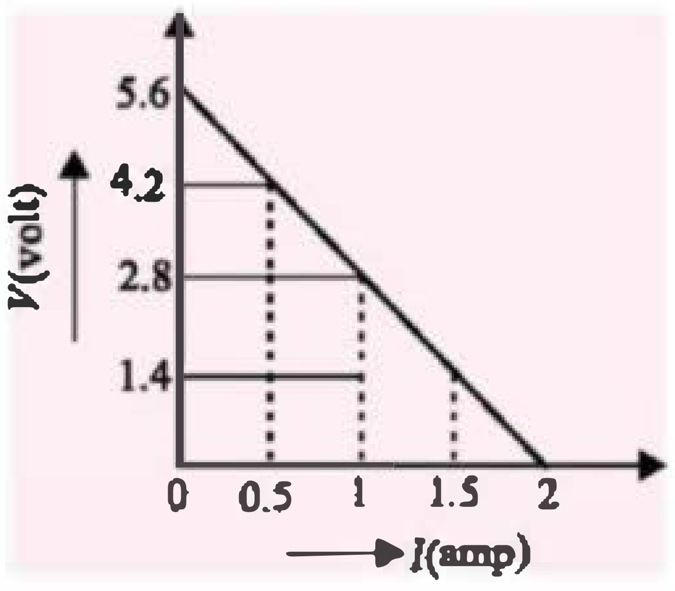
(a) 3.25 A (b) 2.75 A (c) 1.56 A (d) 0.75 A

13. In the circuit shown in figure, when the input voltage of the base resistance is 10 V, V_{BE} is zero and V_{CE} is also zero. Then current amplification factor of transistor is



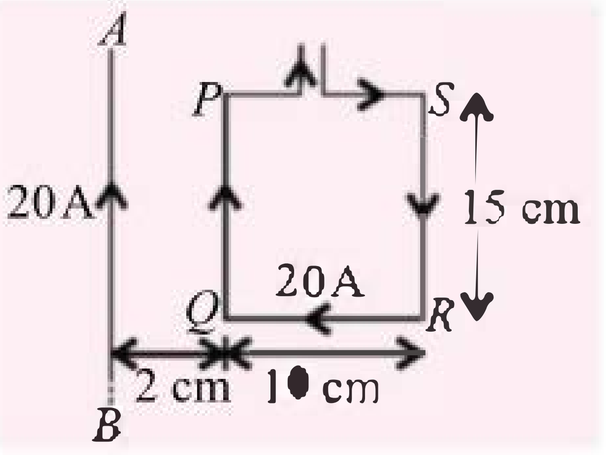
(a) 103 (b) 83 (c) 133 (d) 93

14. Four cells of identical emf E and internal resistance r are connected in series to a variable resistor. The given graph shows the variation of terminal voltage of the combination with current. The emf of each cell used is



(a) 1.4 V (b) 5.6 V
(c) 2 V (d) 1 V

15. The resultant force on the current loop $PQRS$ due to a long current carrying conductor will be



(a) 1.8×10^{-4} N
(b) 5×10^{-4} N
(c) 10^{-4} N
(d) 3.6×10^{-4} N

SOLUTIONS

1. (c) : $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

$\dfrac{1}{v} - \dfrac{\mu_1}{(-u)} = \dfrac{1-\mu_1}{(-R)} \Rightarrow \dfrac{1}{v} + \dfrac{\mu_1}{u} = \dfrac{-1+\mu_1}{R}$

For v to be positive $\dfrac{-1+\mu_1}{R} > \dfrac{\mu_1}{u}$

$\dfrac{1}{\mu_1 R} > \dfrac{1}{u(\mu_1 - 1)} \Rightarrow u > \dfrac{\mu_1}{(\mu_1 - 1)} R$

Comparing with given value, $u > \dfrac{5}{2} R$

$\dfrac{\mu_1}{(\mu_1 - 1)} = \dfrac{5}{2}$ or $\mu_1 = \dfrac{5}{3}$

2. (c) : Average power transferred per unit area = magnitude of poynting vector

$$S = \frac{E_{rms} B_{rms}}{\mu_0} \quad \text{...(i)}$$

$E_{rms} = \dfrac{E_{max}}{\sqrt{2}} = \dfrac{E_0}{\sqrt{2}}, B_{rms} = \dfrac{B_0}{\sqrt{2}}, \dfrac{E_0}{B_0} = c$ and $c = \dfrac{1}{\sqrt{\mu_0 E_0}}$

$\therefore \; E_0 = \sqrt{\dfrac{2S}{\varepsilon_0 c}} = \sqrt{\dfrac{2 \times 1350}{8.85 \times 10^{-12} \times 3 \times 10^8}}$ (using (i))

$\therefore \; E_0 = 1.01 \times 10^3$ V m^{-1}

and $B_0 = \dfrac{E_0}{c} = \dfrac{1.01 \times 10^3}{3 \times 10^8} = 3.37 \times 10^{-6}$ T

3. (c) : For Paschen series

$$\bar{v} = \frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n^2}\right]; n = 4, 5, 6.....$$

For first member of Paschen series, $n = 4$

$$\frac{1}{\lambda_1} = R\left[\frac{1}{3^2} - \frac{1}{4^2}\right] \Rightarrow \frac{1}{\lambda_1} = \frac{7R}{144}$$

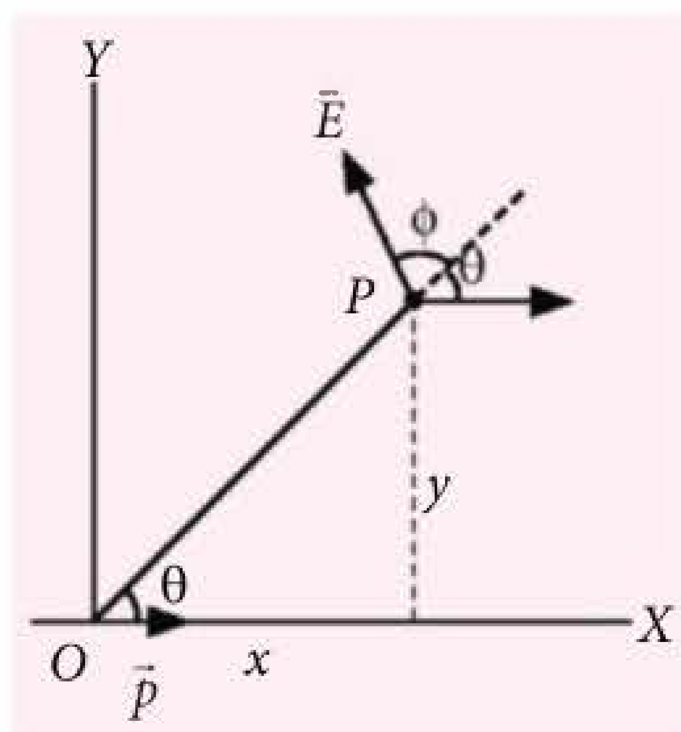
$$\Rightarrow R = \frac{144}{7\lambda_1} = \frac{144}{7 \times 18800 \times 10^{-10}} = 1.1 \times 10^7$$

For shortest wavelength, $n = \infty$

$$\text{So } \frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{\infty^2}\right] = \frac{R}{9}$$

$$\Rightarrow \lambda = \frac{9}{R} = \frac{9}{1.1 \times 10^7} = 8.182 \times 10^{-7} \text{ m} = 8182 \text{ Å}$$

4. (d): Let $P(2, 2\sqrt{2}, 0)$ be the given point. As is clear from given figure, $x = 2, y = 2\sqrt{2}, z = 0$



$$\tan\theta = \frac{y}{x} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\sqrt{2}}$$

Let the electric field $\vec{E}$ due to the dipole be at an angle ϕ with OP.

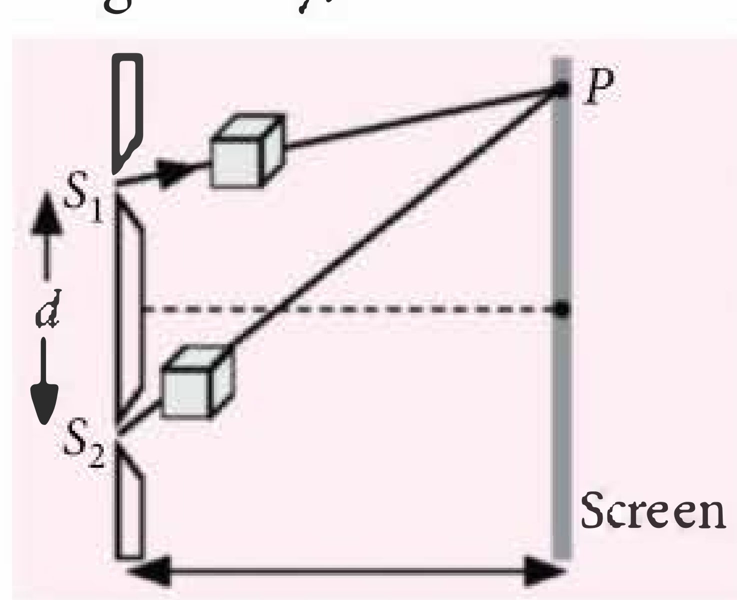
$$\text{As } \tan\phi = \frac{1}{2}\tan\theta = \frac{1}{2}.\sqrt{2} = \frac{1}{\sqrt{2}} = \cot\theta$$

$\therefore \quad \phi = 90° - \theta$

It shows that, $\vec{E}$ is along positive Y-axis.

5. (a): Shift in fringe width is given by,

$$\Delta x = \frac{\beta}{\lambda}(\mu - 1)t$$



Shift due to one plate

$$\Delta x_1 = \frac{\beta}{\lambda}(\mu_1 - 1)t$$

Shift due to another plate, $\Delta x_2 = \frac{\beta}{\lambda}(\mu_2 - 1)t$

Net shift, $\Delta x = \Delta x_2 - \Delta x_1 = \frac{\beta}{\lambda}(\mu_2 - \mu_1)t$ ...(i)

Also it is given that $\Delta x = 5\beta$ ...(ii)

Using eqns. (i) and (ii), $5\beta = \frac{\beta}{\lambda}(\mu_2 - \mu_1)t$

$$\Rightarrow t = \frac{5\lambda}{(\mu_2 - \mu_1)} = \frac{5 \times 4800 \times 10^{-10}}{(1.7 - 1.4)}$$

$$= 8 \times 10^{-6} \text{ m} = 8 \text{ μm}$$

6. (a): Equivalent resistance of 2 Ω and 3 Ω in parallel will be $R_p = \frac{2 \times 3}{2 + 3} = \frac{6}{5} = 1.2\ \Omega$

Since capacitor provides infinite resistance to direct current, hence no current flows in the capacitor arm. Therefore, total resistance of current carrying arms, $R = 1.2 + 2.8 = 4\ \Omega$

Hence current from battery will be, $I = \frac{6}{4} = 1.5$ A

$\therefore$ Potential difference across A and $B = IR_p$
$= 1.5 \times 1.2 = 1.8$ V

$\therefore$ Current through 2 Ω resistor

$$= \frac{\text{potential difference}}{\text{resistance}} = \frac{1.8}{2} = 0.9 \text{ A}$$

7. (b): Equivalent resistance between Q and S,

$$R' = \frac{5 \times 10}{5 + 10} = \frac{10}{3}\Omega$$

Potential difference across Q and S,

$$V_Q - V_S = \frac{2 \times 10}{3} = \frac{20}{3} \text{ V}$$

Current through arm QPS, $I_1 = \frac{20/3}{5} = \frac{4}{3}$ A

Potential difference across Q and P,

$$V_Q - V_P = \frac{4}{3} \times 2 = \frac{8}{3} \text{ V}$$

Current through arm QRS, $I_2 = \frac{20/3}{10} = \frac{2}{3}$ A

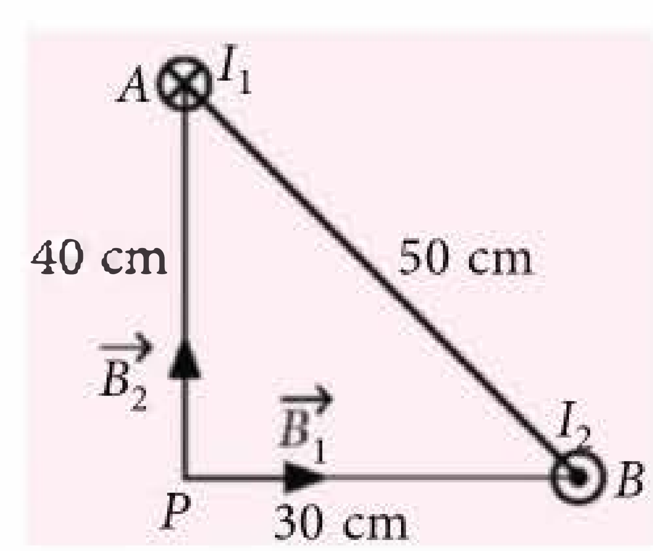
Potential difference across Q and R,

$$V_Q - V_R = \frac{2}{3} \times 3 = 2 \text{ V}$$

$$\therefore V_P - V_R = (V_Q - V_R) - (V_Q - V_P)$$

$$= 2 - \frac{8}{3} = \frac{-2}{3} \approx -0.67 \text{ V}$$

8. (b): From figure, we have $AB = 50$ cm, $AP = 40$ cm and $PB = 30$ cm.



$\because \quad AB^2 = (AP)^2 + (PB)^2$

$\Rightarrow \quad 50^2 = 40^2 + 30^2$

Thus, $\angle APB = 90°$.

Magnetic field induction at P due to downward current I_1 through conductor A is

$$B_1 = \frac{\mu_0}{4\pi}\frac{2I_1}{r_1} = 10^{-7} \times \frac{2 \times 20}{0.40} = 10^{-5} \text{ T along } PB$$

Magnetic field induction at P due to upward current I_2 through conductor B is

$$B_2 = \frac{\mu_0}{4\pi}\frac{2I_2}{r_2} = 10^{-7} \times \frac{2 \times 30}{0.30} = 2 \times 10^{-5} \text{ T along } PA$$

Net magnetic field induction at P is

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(10^{-5})^2 + (2 \times 10^{-5})^2} = \sqrt{5} \times 10^{-5} \text{ T}$$

9. **(b)**: $N = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}}$

Number of atoms at $t = 2$ hr,

$$N_1 = 8\times10^{10}\left(\frac{1}{2}\right)^{2/1} = 2\times10^{10}$$

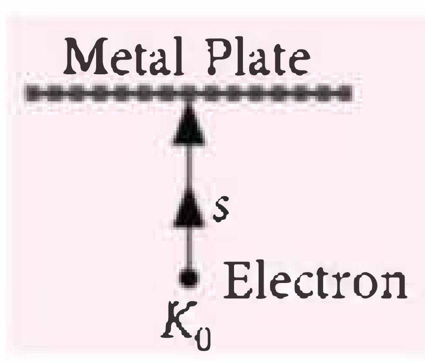
Number of atoms at $t = 4$ hr,

$$N_2 = 8\times10^{10}\left(\frac{1}{2}\right)^{4/1} = \frac{1}{2}\times10^{10}$$

$\therefore$ Number of atoms decayed in given duration

$$= \left(2-\frac{1}{2}\right)\times10^{10} = 1.5\times10^{10}$$

10. **(d)**: Here, $\sigma = -2.0\times10^{-6}$ C m^{-2}



Initial K.E $= K_0 = 100$ eV

$= 100\times1.6\times10^{-19}$ J

$= 1.6\times10^{-17}$ J

Let the electron be fired from a distance s from the plate, as shown in the given figure.

From work energy theorem, work done

$= F\times s =$ loss in K.E

$(-eE)\, s = K_0$ ...(i)

Now, electric field due to the plate (having some thickness), $E = \sigma/\varepsilon_0$

$$\therefore\ -e\frac{\sigma}{\varepsilon_0}s = K_0 \quad \text{(Using (i))}$$

or $$s = \frac{\varepsilon_0 K_0}{-e\sigma} = \frac{8.85\times10^{-12}\times1.6\times10^{-17}}{-1.6\times10^{-19}(-2\times10^{-6})}$$

$= 4.43\times10^{-4}$ m.

11. **(a)**: Here $N_1 = 500$, $N_2 = 50$, $r = 2$ cm $= 0.02$ m

$l = 50$ cm $= 0.50$ m, $\mu_0 = 4\pi\times10^{-7}$ T m A^{-1}

The mutual inductance of the two coils is

$$M = \frac{\mu_0 N_1 N_2 A}{l} = \frac{\mu_0 N_1 N_2 .\pi r^2}{l}$$

$$M = \frac{4\pi\times10^{-7}\times500\times50\times\pi\times(0.02)^2}{0.5}$$

$= 78.87\times10^{-6}$ H $= 78.87$ μH.

The emf induced in the second coil is

$$\varepsilon = -M\frac{dI}{dt} = -78.87\times10^{-6}.\frac{(5-0)}{0.02}$$

$= -19.72\times10^{-3}$ V $= -19.72$ mV

The negative sign indicates a back emf.

12. **(c)**: Alternating emf, $\varepsilon = 220 \sin 100\,\pi t$

Comparing with $\varepsilon = \varepsilon_0 \sin 2\pi\upsilon t$, we get

$\varepsilon_0 = 220$ V and $\upsilon = 50$ Hz

Current amplitude,

$$I_0 = \frac{\varepsilon_0}{\omega L} = \frac{\varepsilon_0}{2\pi\upsilon L} = \frac{220}{2\pi\times50\times1/\pi} = 2.2 \text{ A}$$

The ac ammeter will read the rms value of current.

$$\therefore\ I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{2.2}{\sqrt{2}} = 1.56 \text{ A.}$$

13. **(c)**: Here $V_i = 10$ V, $V_{BE} = 0$, $V_{CE} = 0$, $V_{CC} = 10$ V.

$R_B = 400$ kΩ $= 400\times10^3$ Ω, $R_C = 3$ kΩ $= 3\times10^3$ Ω

Now, $V_i - V_{BE} = R_B I_B$

$\therefore$ $10 - 0 = (400\times10^3)\, I_B$

or $$I_B = \frac{10}{400\times10^3} = 25\times10^{-6} \text{ A}$$

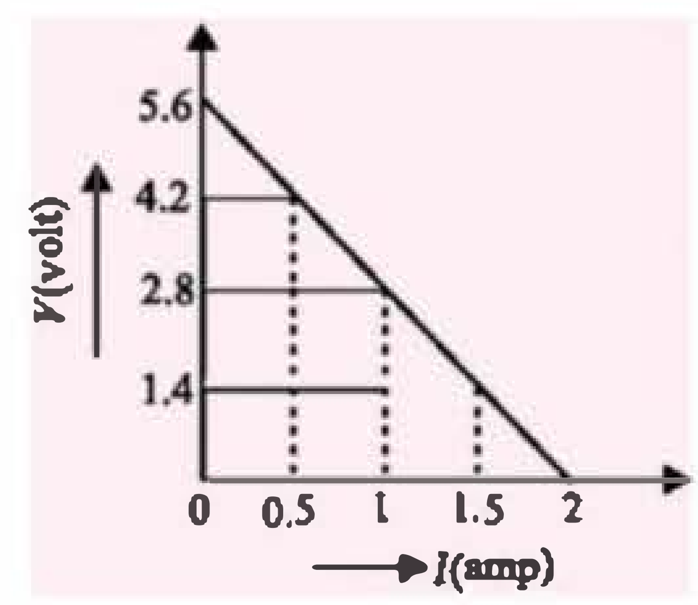
Also, $V_{CC} - V_{CE} = I_C R_C$ or $10 - 0 = I_C\times3\times10^3$

or $$I_C = \frac{10}{3\times10^3} = 3.33\times10^{-3} \text{ A}$$

Amplification factor, $$\beta = \frac{I_C}{I_B} = \frac{3.33\times10^{-3}}{25\times10^{-6}} \simeq 133$$

14. **(a)**: Terminal voltage,

$V = 4E - I(4r)$ ...(i)



From the graph

When $I = 0$, $V = 5.6$ V

$\therefore$ From eqn. (i),

$5.6 = 4E - 0$

$$E = \frac{5.6}{4} = 1.4 \text{ V}$$

15. **(b)**: The effective force is only on PQ and RS.

The force on PQ is attractive and on RS it is repulsive.

Force between two current carrying conductors is F_1 between AB and $PQ = \frac{\mu_0 i_a i_b\cdot L}{2\pi d}$, attractive force and

F_2 between AB and $RS = \frac{\mu_0 i_a i_b\cdot L}{2\pi d'}$, repulsive force

$$\therefore\ F_1 - F_2 = \left(\frac{2\mu_0}{4\pi}\right)\frac{20\times20\times0.15}{0.02} - \frac{2\mu_0}{4\pi}\frac{20\times20\times0.15}{0.12}$$

$$\Rightarrow F_{resultant} = 2\times\frac{\mu_0}{4\pi}\times20\times20\times0.15\cdot\left\{\frac{100}{2} - \frac{100}{12}\right\}$$

$= 5\times10^{-4}$ N.

Monthly Test Drive CLASS XI ANSWER KEY

1. (b)	2. (c)	3. (c)	4. (a)	5. (d)
6. (b)	7. (a)	8. (a)	9. (b)	10. (b)
11. (b)	12. (c)	13. (b)	14. (a)	15. (b)
16. (d)	17. (c)	18. (b)	19. (d)	20. (a,b,c)
21. (a,c)	22. (a,b,d)	23. (a,c)	24. (a,d)	25. (a,b)
26. (2.7)	27. (4)	28. (3)	29. (a)	30. (d)

CLASS-XII

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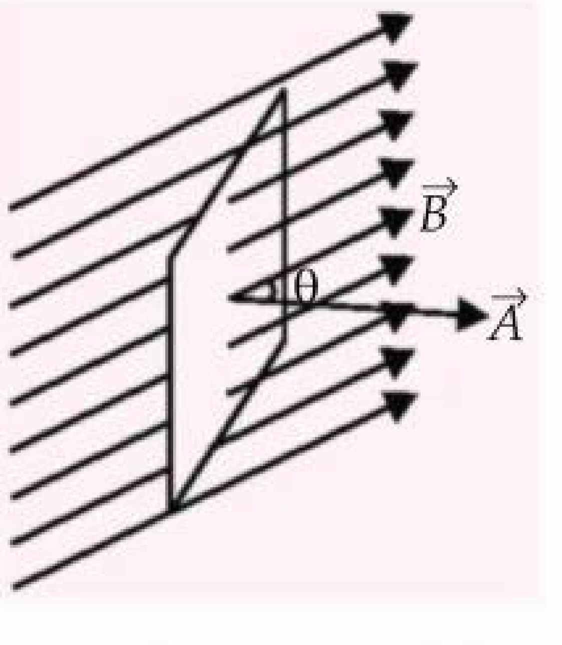
Unit 4 Electromagnetic Induction | Alternating Current

ELECTROMAGNETIC INDUCTION

Electromagnetic induction is the phenomenon of production of emf in a coil, when the magnetic flux linked with the coil is changed. The emf so produced is called induced emf and the current so produced is called induced current.

Magnetic Flux (ϕ)



- The total number of magnetic field lines crossing normally through a surface placed in a magnetic field is called magnetic flux (ϕ) linked with the surface.
  $\phi = \vec{B}.\vec{A} = BA\cos\theta$,
  where B is the magnetic field, A is the area of the surface and θ is the angle between the direction of the magnetic field and normal to the surface.
- The SI unit of magnetic flux is weber (Wb).
- Magnetic flux can be changed by
  - changing the intensity of the magnetic field.
  - changing the orientation of coil with respect to the magnetic field.
  - changing the area of the closed circuit.

Faraday's Laws of Electromagnetic Induction

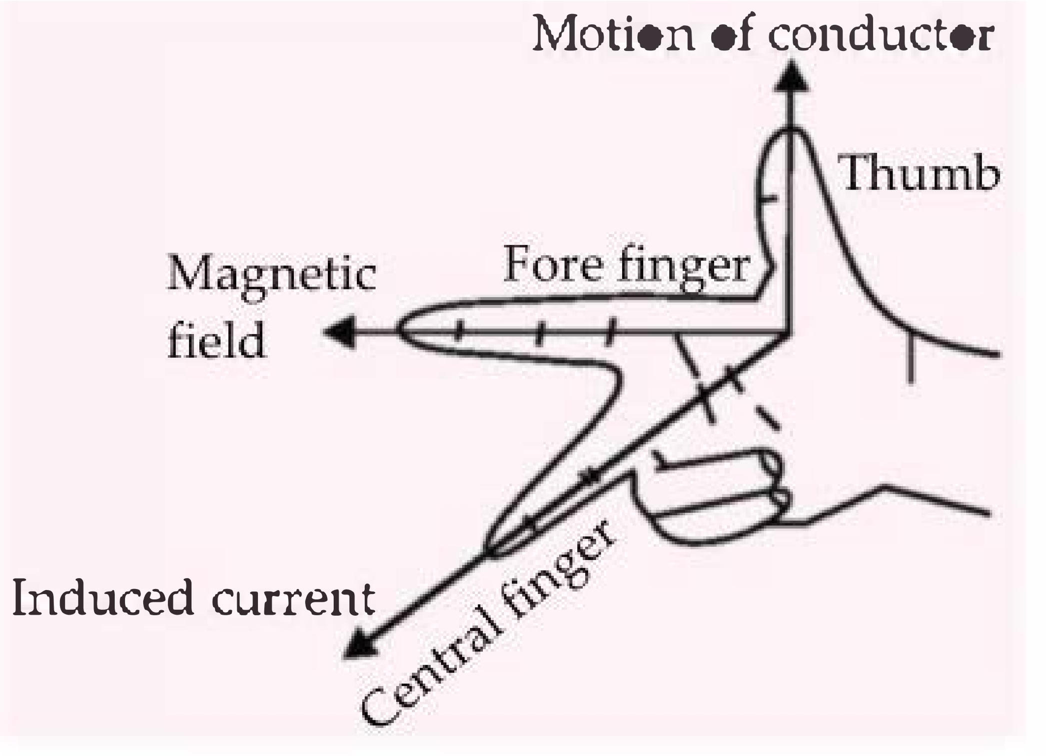
- **First law:** Whenever magnetic flux linked with a circuit (a loop of wire or a coil or an electric circuit in general) changes, induced emf is produced. The induced emf lasts as long as the change in the magnetic flux continues.
- **Second law:** The magnitude of the induced emf is directly proportional to the rate of change of the magnetic flux.
  Induced emf, $\varepsilon = -\frac{d\phi}{dt} = -\frac{\phi_2 - \phi_1}{t}$

Lenz's Law

- It states that the induced current produced in a circuit always flows in such a direction that it opposes the change or the cause that produces it.
- Lenz's law can be used to find the direction of induced current. Lenz's law is in accordance with the law of conservation of energy.

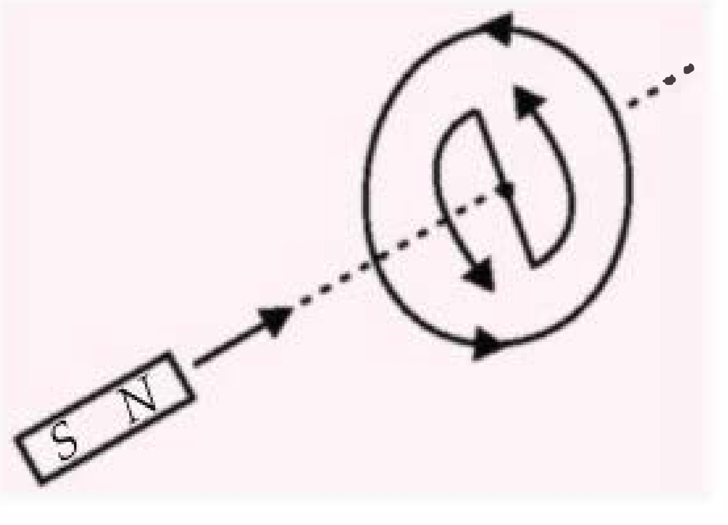
Fleming's Right Hand Rule

- Fleming's right hand rule gives us the direction of induced emf or current, in a conductor moving in a magnetic field.
  According to this rule, if we stretch the fore finger, central finger and thumb of our right hand in mutually perpendicular directions such that fore finger points along the direction of the field and thumb is along the direction of motion of the conductor, then the central finger would give us the direction of induced current or emf.

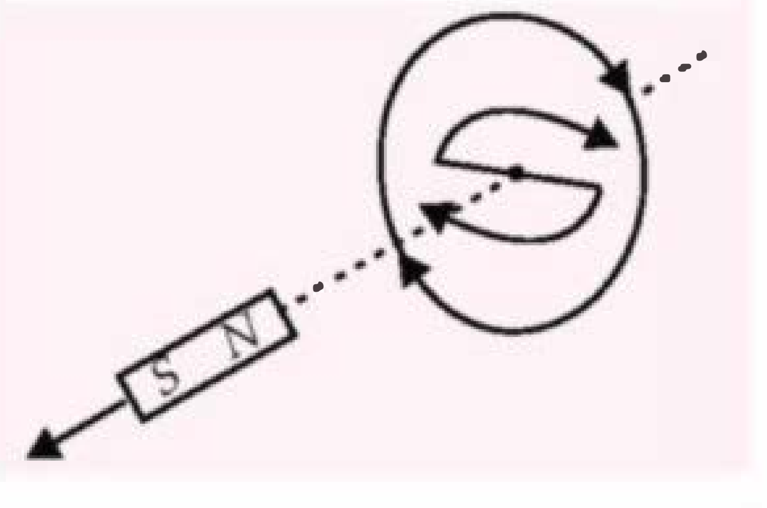


Applications of Lenz's law

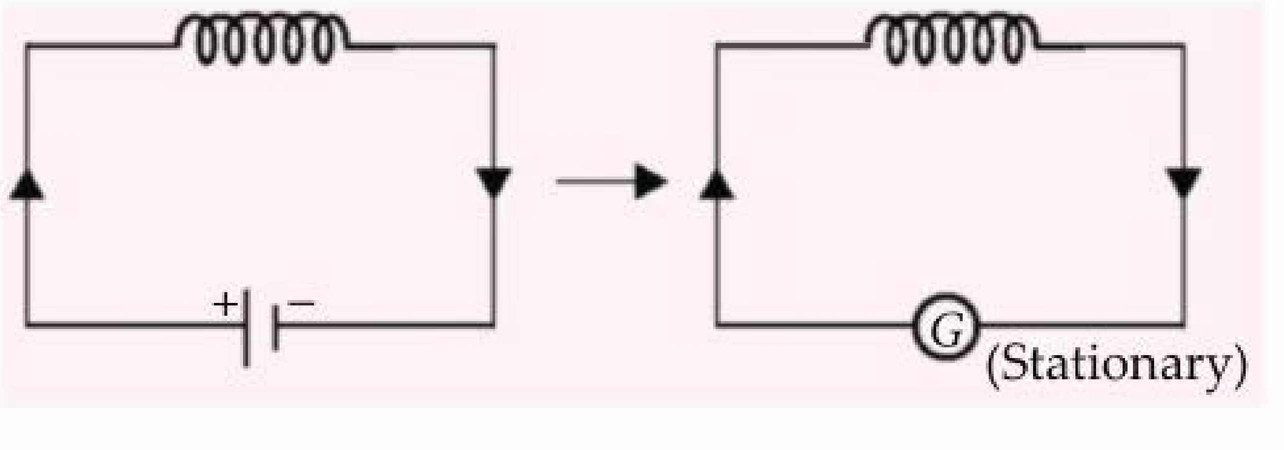
- When the North pole of a bar magnet is moved towards a coil as shown in the figure, the current induced in the coil will be in anticlockwise direction.



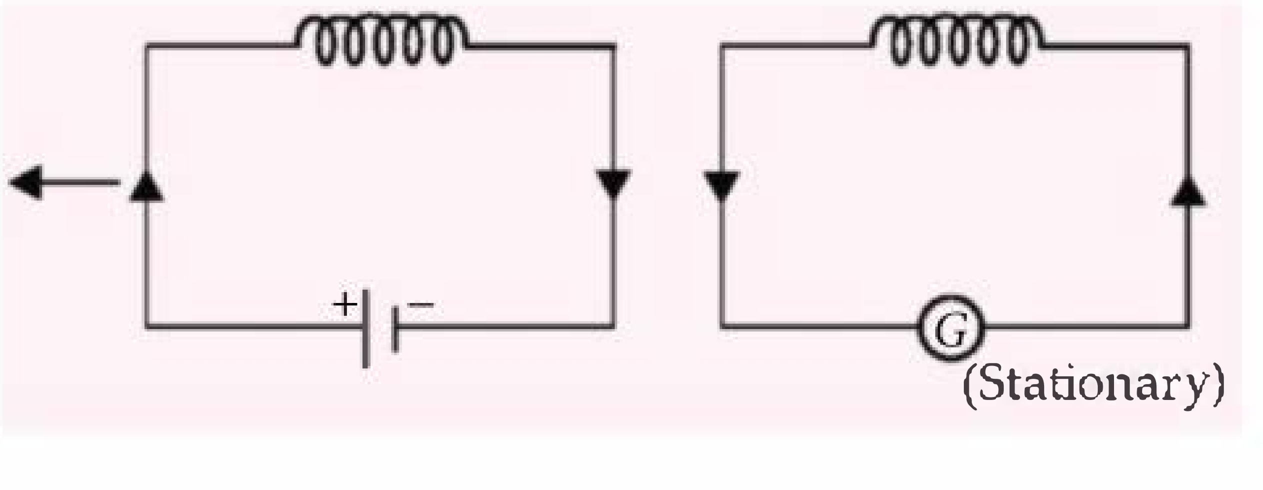
- When the North pole of a bar magnet is moved away from the coil as shown in the figure, the current induced in the coil will be in clockwise direction.



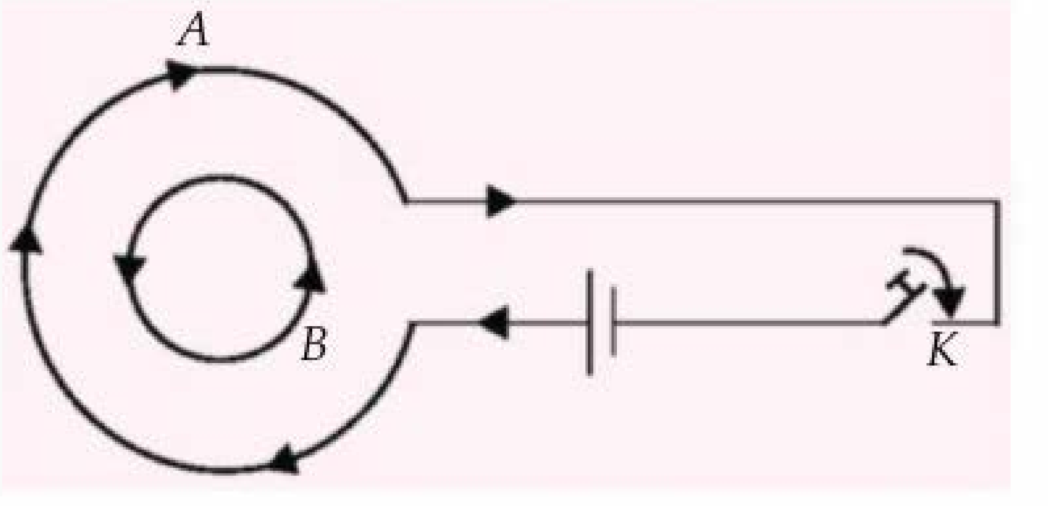
- When a current carrying coil is moved towards a stationary coil, the direction of current induced in stationary coil is as shown in figure below.



- When a current carrying coil is moved away from a stationary coil, the direction of current induced in stationary coil is as shown in figure below.

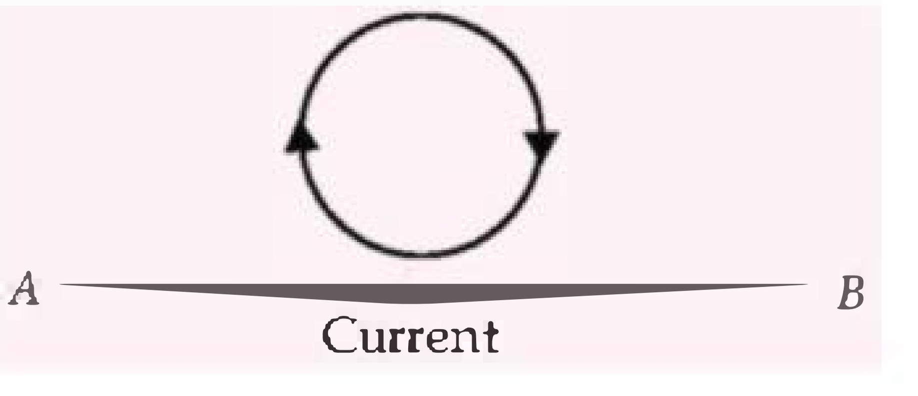


- When two coils A and B are arranged as shown in figure, then on pressing K, current in A increases in clockwise direction. Therefore, induced current in B will be in anticlockwise direction.



However, when key K is released, current in A decreases in clockwise direction. Therefore, induced current in B will be in clockwise direction.

- When current in a straight conductor AB is increased, induced current in loop will be in clockwise direction as shown in the figure. If current in AB is decreasing, the induced current in the loop will be in anticlockwise direction.



Motional Electromotive Force

- If a conductor is moving with velocity $\vec{v}$ in a magnetic field, electrons inside it experience a force $\vec{F} = q(\vec{v} \times \vec{B})$ and accumulate at the end of the conductor. Very soon, an electric field is established. Eventually component of magnetic force along the conductor length is balanced by the electric field force and the drifting of electrons stops and an emf is established.

Now, $\varepsilon = -\oint \vec{E} \cdot d\vec{l} = \int (\vec{v} \times \vec{B}) \cdot d\vec{l} \Rightarrow \varepsilon = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$

This is general expression for induced emf in a conducting wire. If $\vec{v}, \vec{B}$ and l are mutually perpendicular to each other, then $\varepsilon = Bvl$.

- If conducting rod moves on two parallel conducting rails, then phenomenon of induced emf can be understood by the concept of changing area (lvt).

Hence induced emf $|\varepsilon| = \frac{d\phi}{dt} = Bvl$

- **Induced Current :** $I = \frac{\varepsilon}{R} = \frac{Bvl}{R}$

- **Magnetic Force :** $F_m = BIl = B\left(\frac{Bvl}{R}\right)l = \frac{B^2vl^2}{R}$
- **Power dissipated in moving the conductor :**

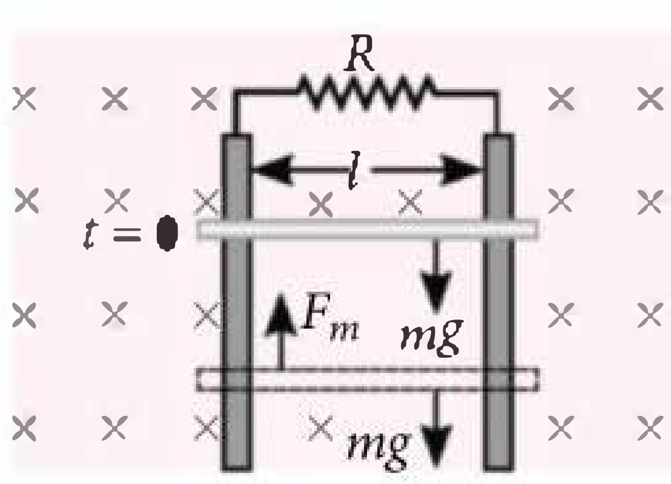
$$P_{mech} = P_{ext} = \frac{dW}{dt} = F_{ext}\cdot v = \frac{B^2vl^2}{R}\times v = \frac{B^2v^2l^2}{R}$$

- **Electrical Power :** $P_{thermal} = \frac{H}{t} = I^2R = \left(\frac{Bvl}{R}\right)^2\cdot R$

$$\therefore\ P_{thermal} = \frac{B^2v^2l^2}{R}$$

- Motion of conducting rod in a vertical plane
  - Rod will achieve a constant maximum (terminal) velocity v_T if $F_m = mg$

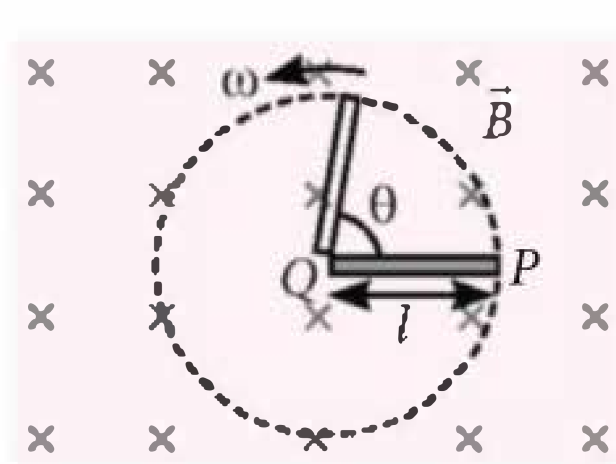
    So, $\frac{B^2v_Tl^2}{R} = mg; v_T = \frac{mgR}{B^2l^2}$



- Motional emf due to rotational motion
  - emf induced across the ends of the rod is,

    $\varepsilon = \frac{1}{2}Bl^2\omega = Bl^2\pi\upsilon$

    $= \frac{Bl^2\pi}{T}$



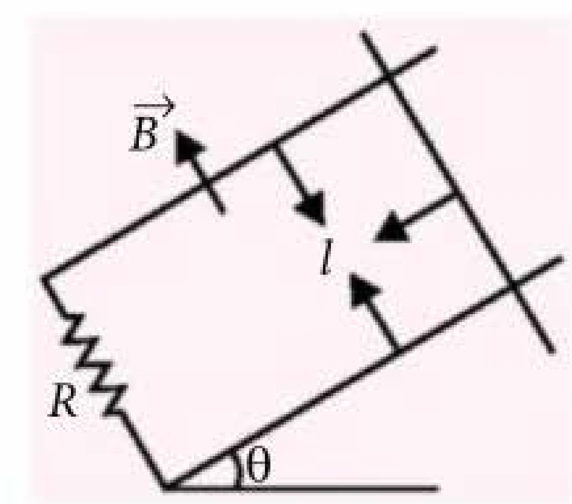
Eddy Currents

- The currents induced in the body of a conductor, when the magnetic flux linked with the conductor changes are called eddy currents.
- The direction of the eddy currents in the conductor can be found by applying Lenz's law or Fleming's right hand rule.
- **Applications of eddy currents**
  - Induction furnace is based on the heating effect of eddy currents.
  - Speedometer is a device used to measure the instantaneous speed of a vehicle.
  - Concept of eddy current is used in energy meter to record the consumption of electricity.

Peep Into Previous Years

1. A 10 m long horizontal wire extends from North East to South West. It is falling with a speed of 5.0 m s^{-1}, at right angles to the horizontal component of the earth's magnetic field of 0.3×10^{-4} Wb m^{-2}. The value of the induced emf in wire is
   (a) 0.3×10^{-3} V (b) 2.5×10^{-3} V
   (c) 1.5×10^{-3} V (d) 1.1×10^{-3} V
   (JEE Main 2019)

2. A copper rod of mass m slides under gravity on two smooth parallel rails, with separation l and set at an angle of θ with the horizontal. At the bottom, rails are joined by a resistance R. There is a uniform magnetic field B normal to the plane of the rails, as shown in the figure. The terminal speed of the copper rod is



   (a) $\frac{mgR\sin\theta}{B^2l^2}$ (b) $\frac{mgR\cot\theta}{B^2l^2}$
   (c) $\frac{mgR\tan\theta}{B^2l^2}$ (d) $\frac{mgR\cos\theta}{B^2l^2}$
   (JEE Main 2018)

3. In which of the following devices, the eddy current effect is not used?
   (a) Electric heater (b) Induction furnace
   (c) Magnetic braking in train
   (d) Electromagnet ***(NEET 2019)***

Inductance

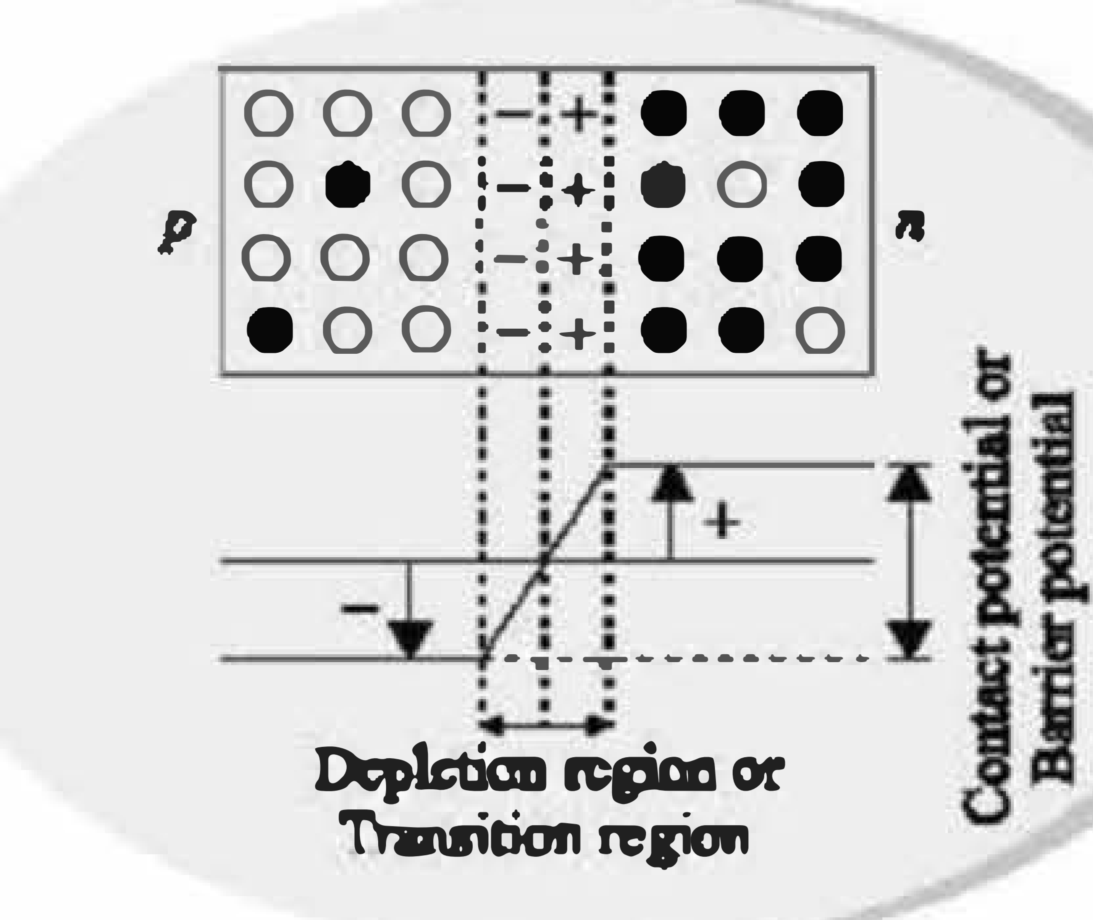
- Inductance the analogous to inertia in mechanics, because inductance of an electrical circuit opposes any change of current in the circuit.
- Inductance is a scalar quantity. It has dimensions of $[ML^2T^{-2}A^{-2}]$. SI unit of inductance is henry.
- **Self Inductance**
  - When the current in a coil changes, it induces a back emf in the same coil. The self-induced emf is given by, $\varepsilon = -L\frac{dI}{dt}$, where L is the self-inductance of the coil.

    $L = \frac{N\phi}{I} = \frac{NBA}{I} = \frac{\phi_{total}}{I}$

    It is a measure of the inertia of the coil against the change of current through it.

BRAIN MAP

ELECTRONIC DEVICES

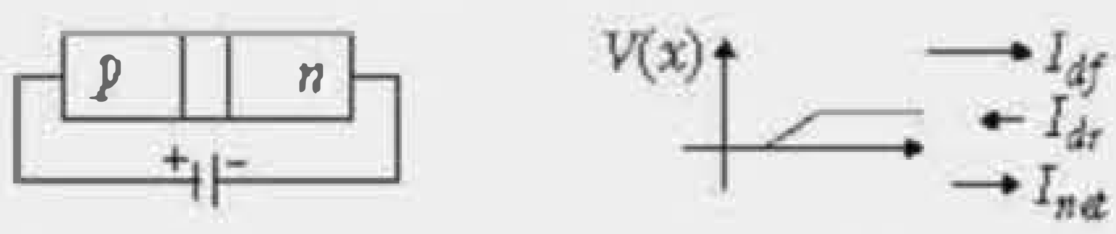


Formation of *p-n* Junction

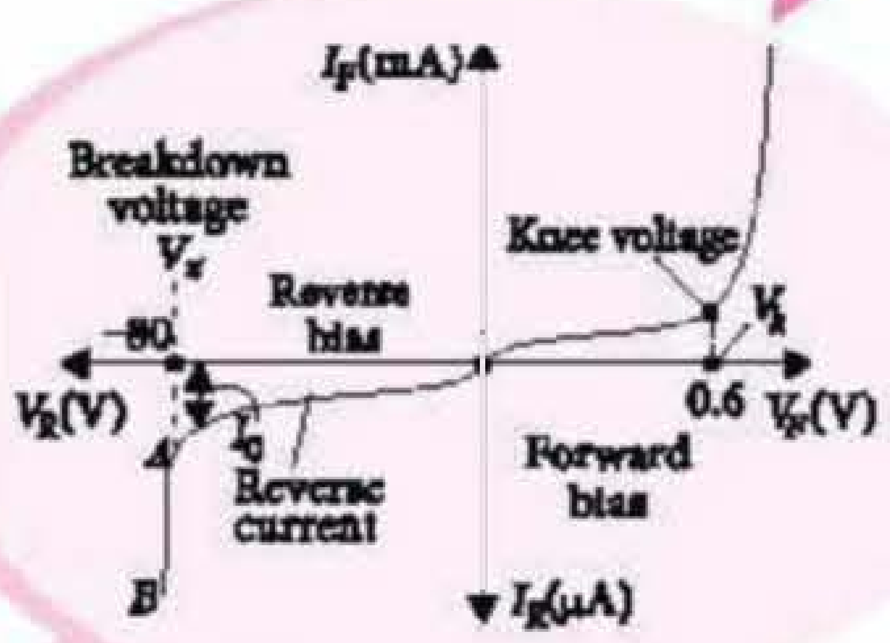
- Due to high carrier concentration difference, holes diffuses from *p*-side to *n*-side and electron diffuses from *n*-side to *p*-side, produces diffusion current.
- Due to barrier potential, majority charge carriers are forced to move to other side of the junction and such movement produces drift current.

Biasing of *p-n* junction diode

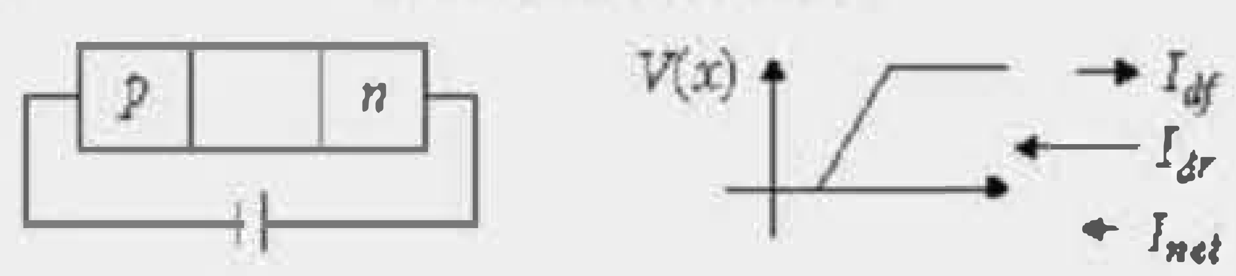
Forward Biased



Effective barrier potential decreases.
Depletion width decreases.
Low resistance offered at junction.
High current flows through the circuit.



Reversed Biased



- Effective barrier potential increases.
- Depletion width increases.
- High resistance offered at junction.
- Low current flows through the circuit.
- Reverse breakdown occurs at a high reverse bias voltage.

V-I Characteristic of a *p-n* Junction Diode

Current flowing through diode,
$I = I_0\,[e^{(eV/nKT)} - 1]$

Dynamic or ac resistance, $r_d = \dfrac{\Delta V}{\Delta I}$

Static or dc resistance, $r_{dc} = \dfrac{V}{I}$

p-n

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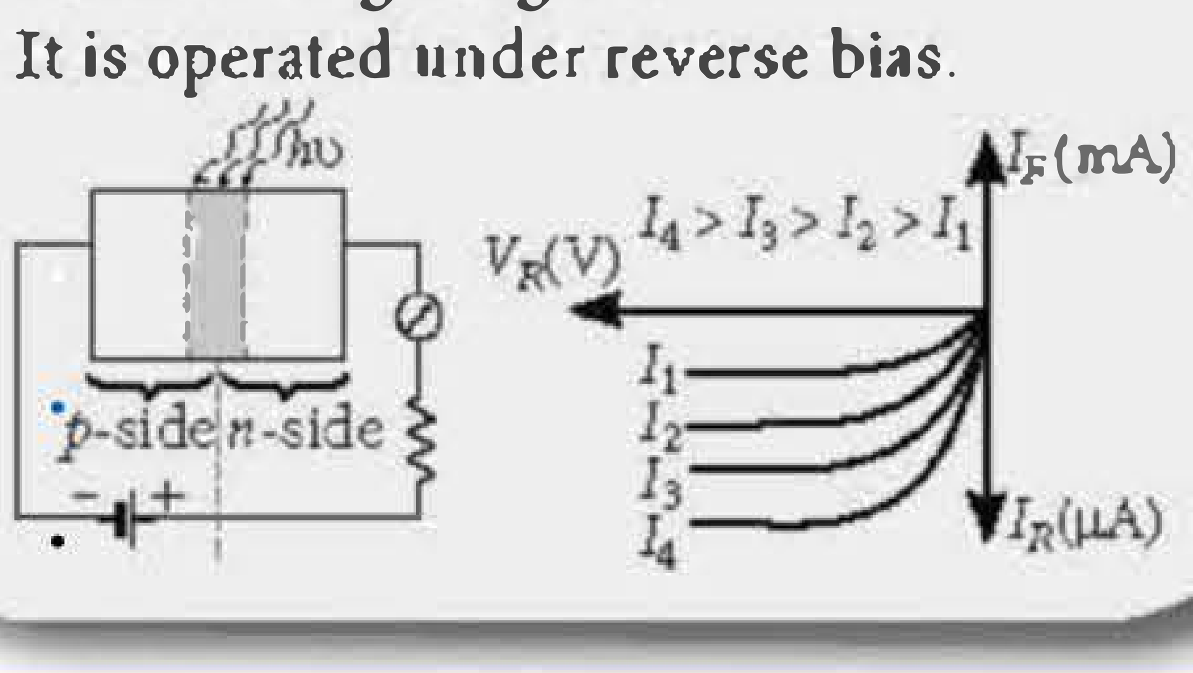
p-n

- The most import of a *p-n* junction conduct current in
- In the reverse direc

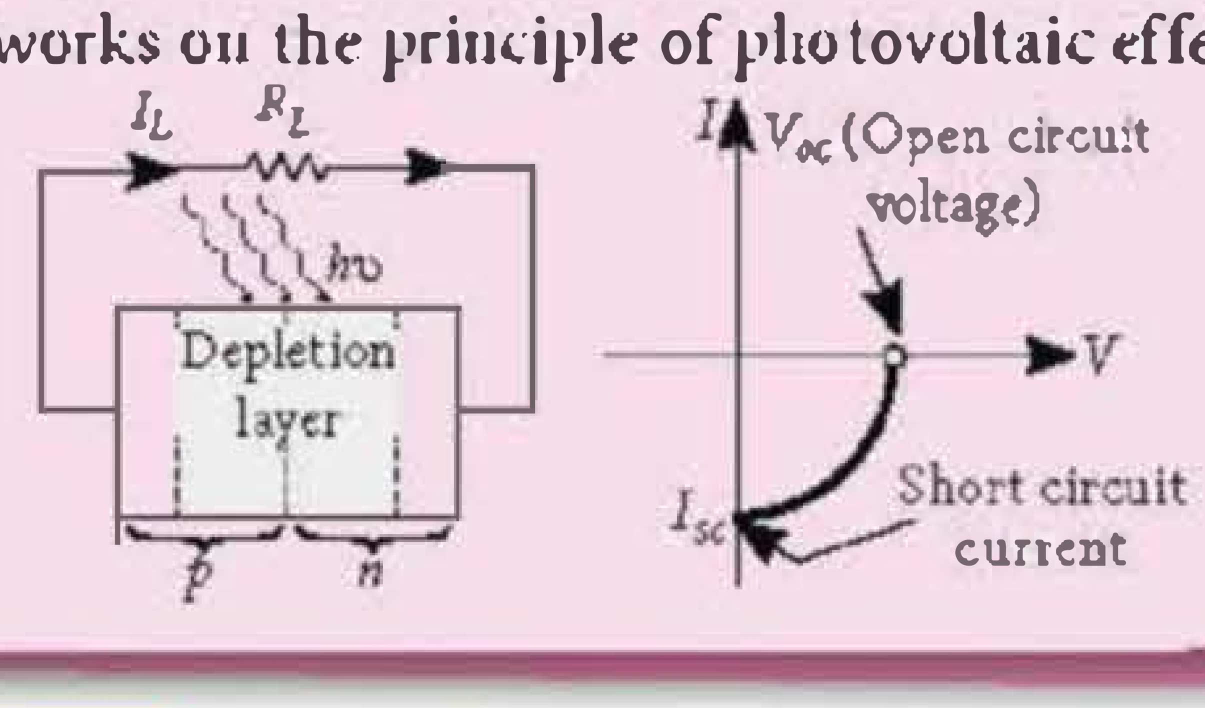
Photodiode

It is fabricated with transparent window to allow light to fall on the diode to detect the light signal.
It is operated under reverse bias.



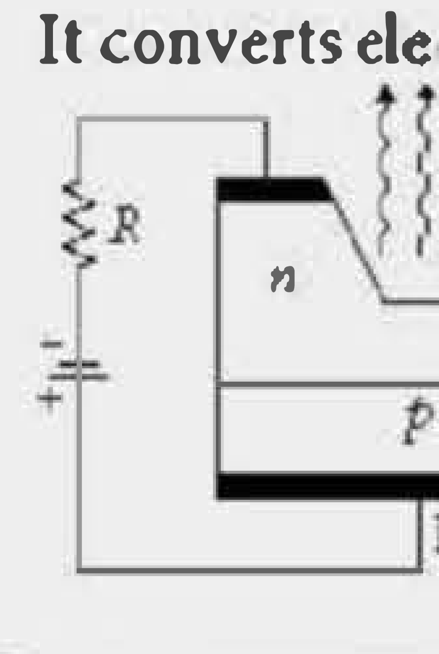
Solar Cell

- Used to convert solar energy into electrical energy.
- It works on the principle of photovoltaic effect.



Ligh

- Used to emit forward bias.
- It converts ele



Transistor

A semiconductor device possessing fundamental action of transfer resistor.

Transistors are of two types :

- ***n-p-n* transistor:** A thin layer of *p*-type semiconductor is sandwiched between two *n*-type semiconductors.
- ***p-n-p* transistor:** A thin layer of *n*-type semiconductor is sandwiched between two *p*-type semiconductors.

There are three configurations of transistors

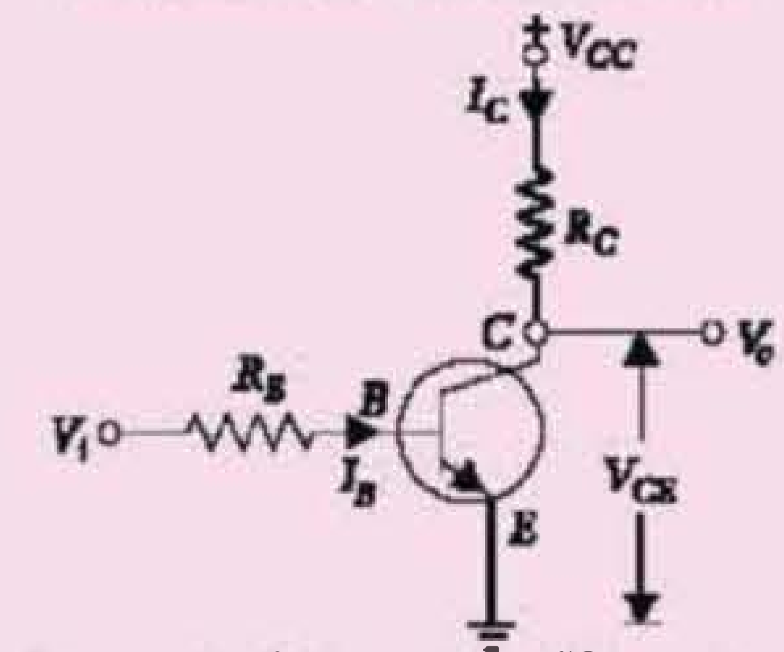
- CB (Common Base), CE (Common Emitter), CC (Common Collector).

Transistor characteristics

- Input resistance, $(r_i)_{(CE)} = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}=\text{constant}}$
- Output resistance, $(r_o)_{(CE)} = \left(\frac{\Delta V_{CE}}{\Delta I_C}\right)_{I_B=\text{constant}}$
- Current amplification factor, $\beta_{ac} = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE}=\text{constant}}$ $\alpha_{ac} = \left(\frac{\Delta I_C}{\Delta I_E}\right)_{V_{CB}=\text{constant}}$

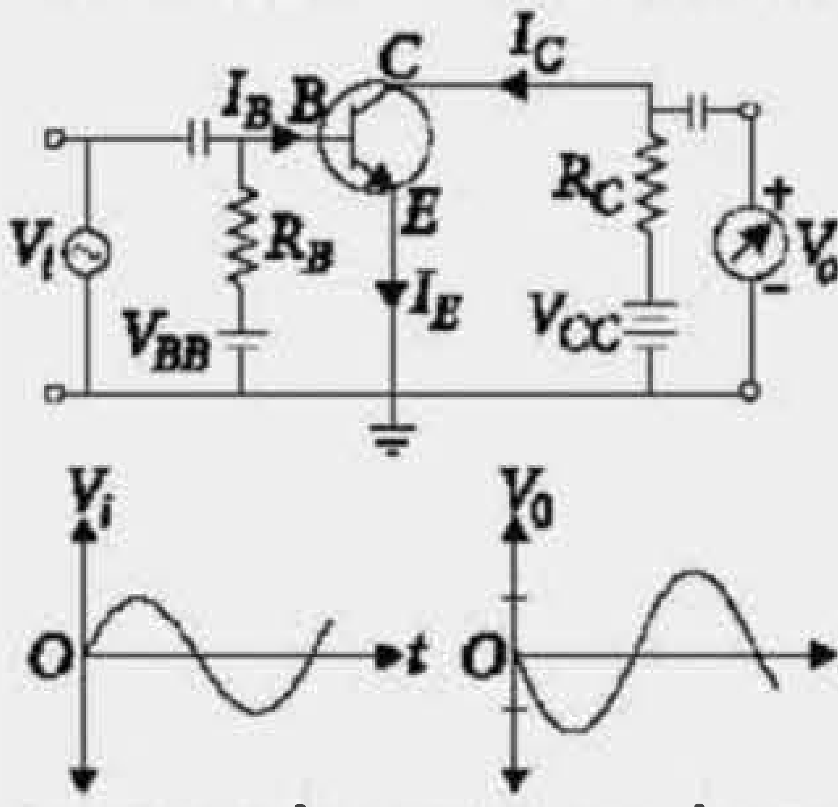
Applications of Transistor

Transistor as a Switch



Operated in cut-off region or saturation region.

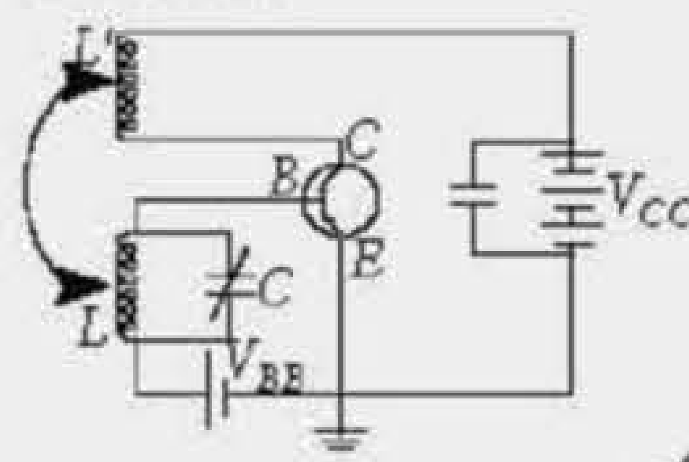
Transistor as an Amplifier



Input and output signals are 180° out of phase.

Transistor as an Oscillator

An oscillator produces a continuing, repeated waveform without input other than perhaps a trigger.



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Diode

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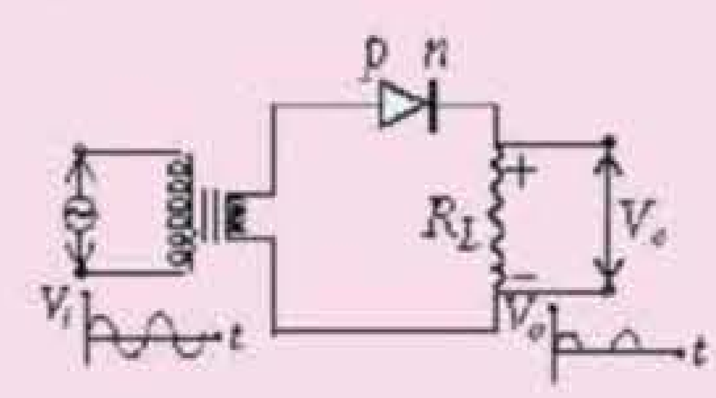
Applications of *p-n* Junction Diode

Rectifier

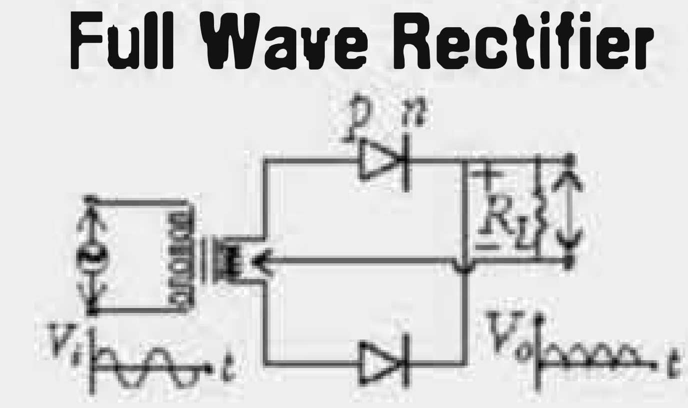
Used to convert ac signal into dc signal.

Half Wave Rectifier



- $I_{max} = \frac{\varepsilon_{max}}{(r_f + R_L)}$; $I_{dc} = \frac{I_{max}}{\pi}$
- Output frequency = Input frequency
- Efficiency = $\frac{P_{dc}}{P_{ac}} = 40.6\%$

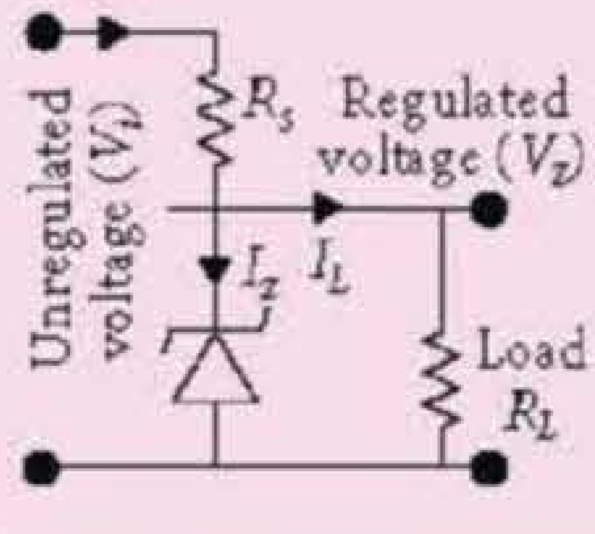
Full Wave Rectifier

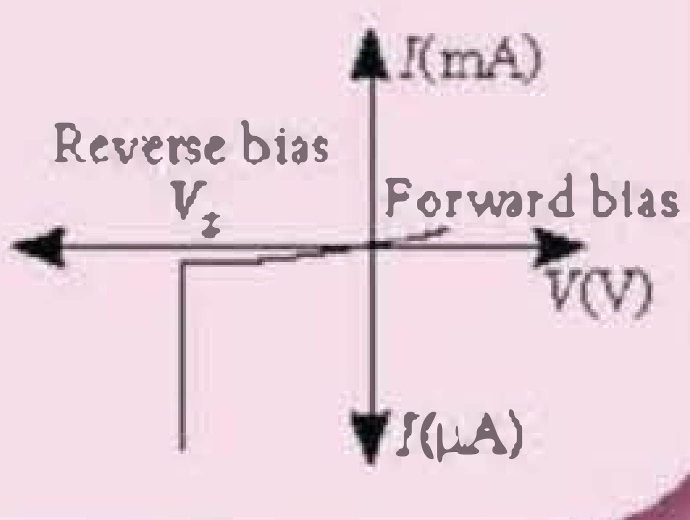


- $I_{max} = \frac{\varepsilon_{max}}{(r_f + R_L)}$; $I_{dc} = \frac{2I_{max}}{\pi}$
- Output frequency = 2 × Input frequenc
- Efficiency = $\frac{P_{dc}}{P_{ac}} = 81.2\%$

Zener Diode

- Used as a voltage regulator.
- Heavily doped *p-n* junction diode.
- Operated under reverse bias condition.





Diode

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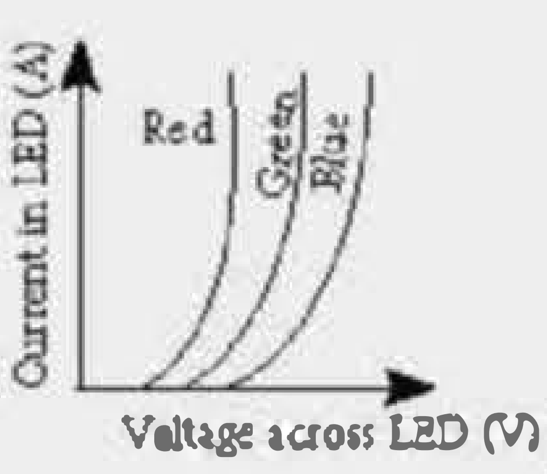
gy into light energy.



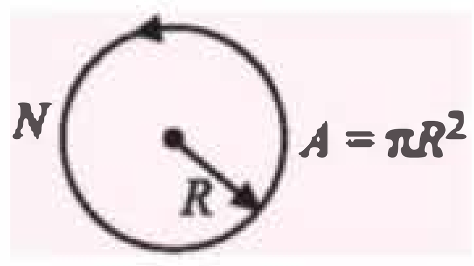
– The self inductance of a long solenoid, the core of which consists of a magnetic material of permeability μ_r is

$$L = \frac{\mu_0 \mu_r N^2 A}{l} = \mu_r \mu_0 \, n^2 \, Al = \mu_0 \, \mu_r \, n^2 \, V$$

Here V = volume of solenoid = Al

- Self inductance of a planar coil of radius R

$$L = \frac{\mu_0 N^2 \pi R}{2}$$



- **Mutual inductance**

– When an emf is produced in a coil because of change in current in a coupled coil, the effect is called mutual inductance.

$$M_{12} = \frac{N_1 \phi_1}{I_2} \text{ and } M_{21} = \frac{N_2 \phi_2}{I_1}$$

– For same length and different number of turns per unit length of two solenoids mutual inductance is given by

$M_{12} = M_{21} = \mu_r \, \mu_0 \, n_1 \, n_2 \, \pi \, r_1^2 \, l = M$

r_1 = radius of inner coil

- Mutual inductance of two concentric and coplanar coils $M_{C_1C_2} = \dfrac{N_2 B_1 A_2}{I_1} = \dfrac{\mu_0 N_1 N_2 \pi r_2^2}{2 r_1}$

- Magnetic energy per unit volume, $u_B = \dfrac{U_B}{V} = \dfrac{B^2}{2\mu_0}$

- **Combination of inductance**

– **Series combination**

$L_S = L_1 + L_2$ (take $M = 0$). If $M \neq 0$ then $L_S = L_1 + L_2 \pm 2\,M$.

The plus sign occurs if windings in the two coils are in the same sense, while minus sign occurs if windings are in opposite sense.

– **Parallel combination,** $\dfrac{1}{L_p} = \dfrac{1}{L_1} + \dfrac{1}{L_2}$

$\therefore \quad L_p = \dfrac{L_1 L_2}{L_1 + L_2}$ (take $M = 0$)

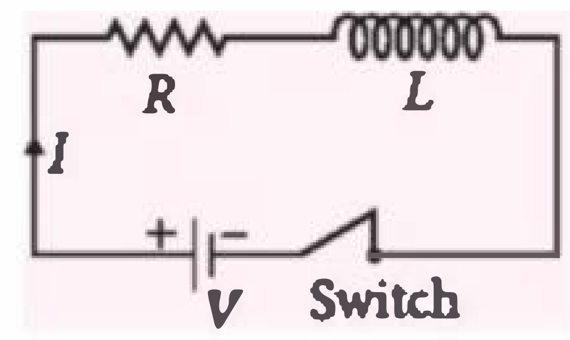
When $M \neq 0$ *i.e.*, they situated close to each other, $L_p = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 \pm M}$

Energy Stored in an Inductor

When a current I flows through an inductor, the energy stored in it, is given by $U_B = \dfrac{1}{2} L I^2$

The energy stored in an inductor is in the form of magnetic energy.

Current Growth in an *LR* Circuit

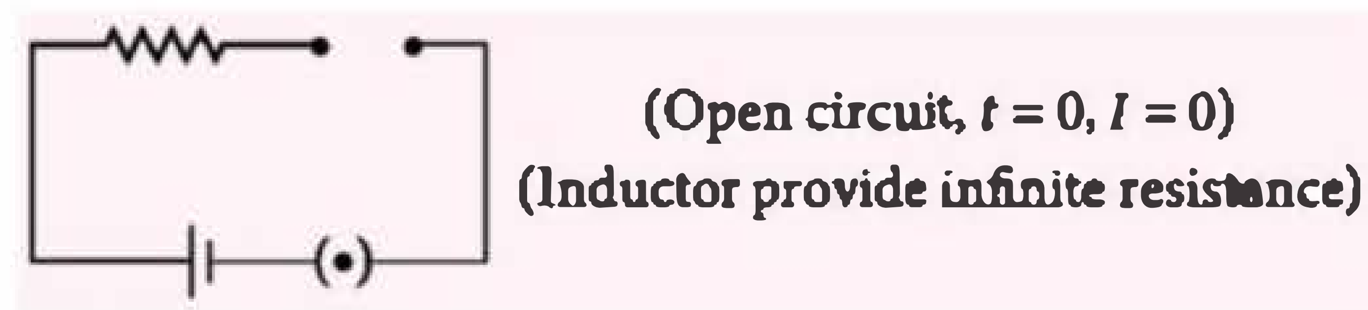


- Emf equation :

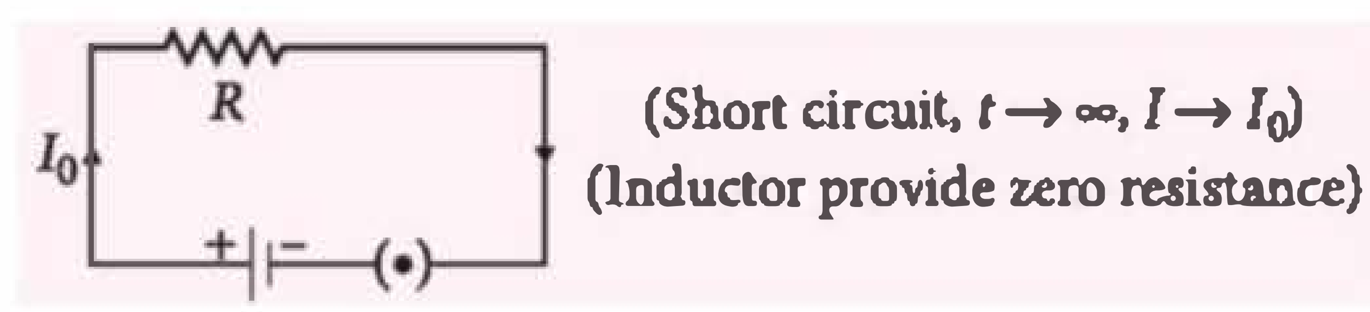
$$V = IR + L\frac{dI}{dt}$$

- Current at any instant : When key is closed the current in circuit increases exponentially with respect to time. The current in circuit at any instant t is given by $I = I_0(1 - e^{-t/\tau})$

- Just after the closing of key, inductance behaves like open circuit and current in circuit is zero.



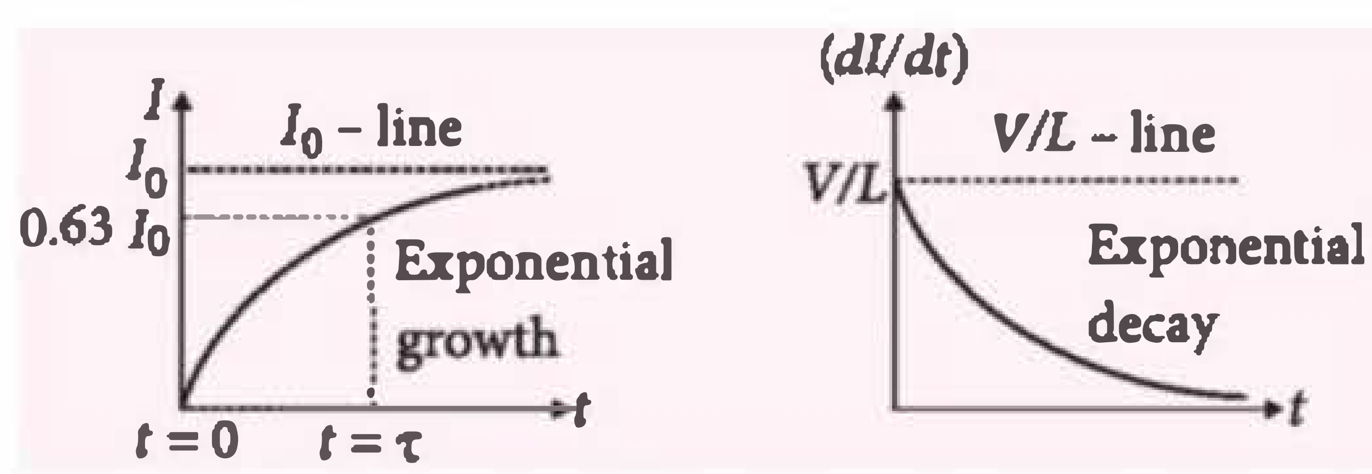
- Some time after closing of the key inductance behaves like short circuit and current in circuit is constant.



– $I_0 = \dfrac{V}{R}$ (maximum or peak value of current)

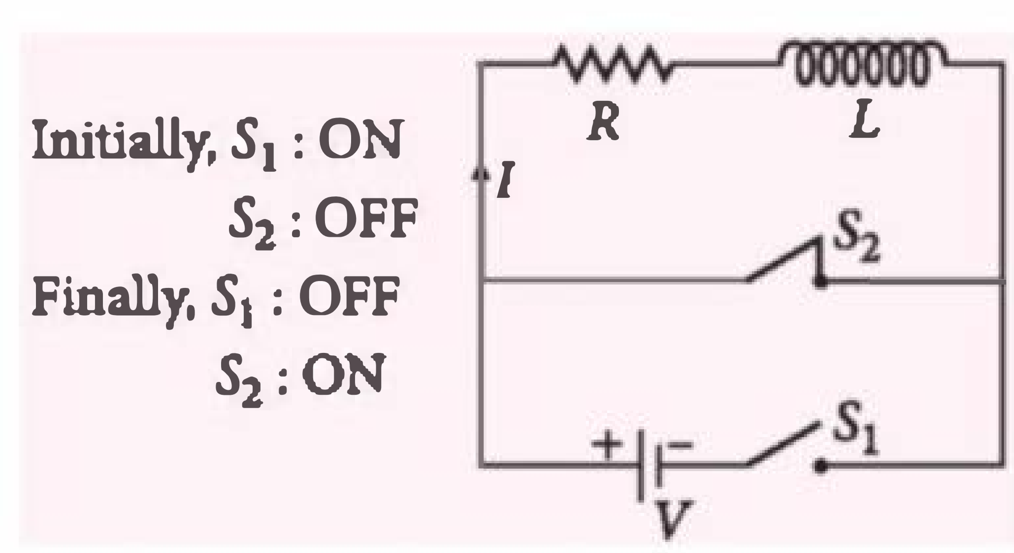
– Peak value of current in circuit does not depend on self inductance of coil.

- Time constant : It is a time in which current increases up to 63% or 0.63 times of peak current value and given as $\tau = L/R$

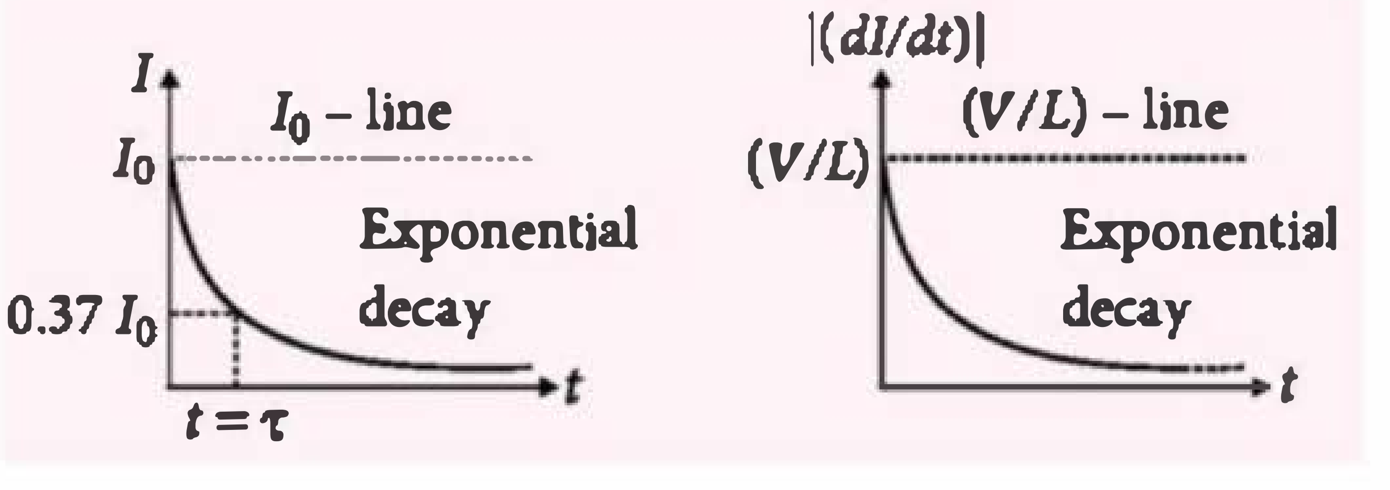


Current Decay in an *LR* Circuit

- Emf equation : $IR + L\dfrac{dI}{dt} = 0$



- Current at any instant : Once current acquires its final maximum steady value, if suddenly required switching positions (S_1 and S_2) are interchanged then current starts decreasing exponentially with respect to time. The current in the circuit at any instant t is given by $I = I_0 e^{-t/\tau}$
  - Just after opening of key $(t = 0) \Rightarrow I = I_0 = \frac{V}{R}$
  - Some time after opening of key $(t \to \infty) \Rightarrow I_0 \to 0$
- Time constant (τ) : It is a time in which current decreases up to 37% or 0.37 times of peak current value $\tau = L/R$



AC Generator

It is a device used to obtain a supply of alternating emf by converting rotational mechanical energy into electrical energy. It is based on the phenomenon of electromagnetic induction. *i.e.* when a coil is rotated in uniform magnetic field, an induced emf is produced in it.

The instantaneous value of the emf produced is given by $\varepsilon = NBA\omega\sin\omega t$, where N is number of turns of the coil, A is the area of coil and ω is angular frequency of rotation of the coil in a magnetic field strength B.

Peep Into Previous Years

4. The self induced emf of a coil is 25 volts. When the current in it is changed at uniform rate from 10 A to 25 A in 1 s, the change in the energy of the inductance is

(a) 637.5 J (b) 540 J
(c) 437.5 J (d) 740 J

(JEE Main 2019)

5. The magnetic potential energy stored in a certain inductor is 25 mJ, when the current in the inductor is 60 mA. This inductor is of inductance

(a) 0.138 H (b) 138.88 H
(c) 1.389 H (d) 13.89 H

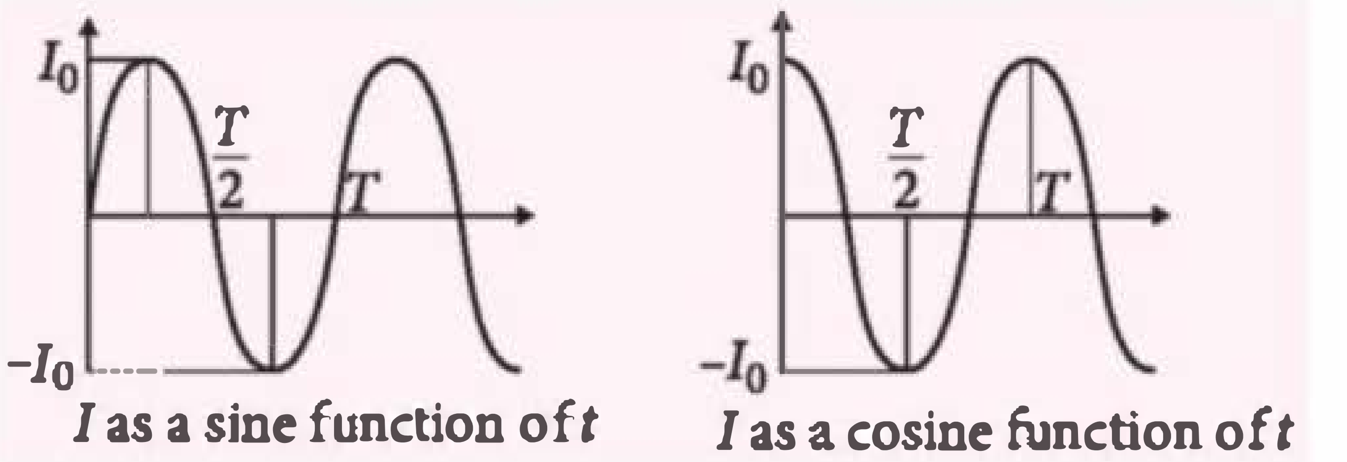
(NEET 2018)

Alternating Current

It is the current which varies continuously in magnitude and periodically in direction. It can be represented by $I = I_0 \sin\omega t$ or $I = I_0\cos\omega t$.

where I_0 is peak value of current and is known as amplitude of ac and I is the instantaneous value of alternating current.

Angular frequency, $\omega = \frac{2\pi}{T} = 2\pi\upsilon$ where T is period of ac and υ is frequency of ac.



I as a sine function of *t* *I* as a cosine function of *t*

Mean or Average Value of AC

Average value of the alternating current over a half cycle is the value of direct current which will sends the same amount of charge in a circuit in a time of half cycle as is sent by the given ac in the same circuit in the same time.

For the complete cycle,

$$I_m \text{ or } \overline{I} \text{ or } I_{av} = \frac{\int_0^T I_0 \sin\omega t \, dt}{\int_0^T dt} = 0$$

$$V_m \text{ or } \overline{V} \text{ or } V_{av} = \frac{\int_0^T V_0 \sin\omega t \, dt}{\int_0^T dt} = 0$$

Average value of alternating current for first half cycle is

$$I_{av} = \frac{\int_0^{T/2} I_0 \sin\omega t \, dt}{\int_0^{T/2} dt} = \frac{2I_0}{\pi} = 0.637 I_0$$

Similarly, for alternating voltage, the average value over first half cycle is

$$V_{av} = \frac{\int_0^{T/2} V_0 \sin\omega t \, dt}{\int_0^{T/2} dt} = \frac{2V_0}{\pi} = 0.637 V_0$$

Average value of alternating current for second cycle is

$$I_{av} = \frac{\int_{T/2}^{T} I_0 \sin \omega t dt}{\int_{T/2}^{T} dt} = -\frac{2I_0}{\pi} = -0.637\, I_0$$

Similarly, for alternating voltage, the average value over second half cycle is

$$V_{av} = \frac{\int_{T/2}^{T} V_0 \sin \omega t dt}{\int_{T/2}^{T} dt} = -\frac{2V_0}{\pi} = -0.637\, V_0$$

Root Mean Square Value of Alternating Current

The value of that direct current which produces heat at the same rate as the alternating current in a given resistor is known as the rms value of alternating current. *rms* value is also known as virtual value or effective value. All ac instruments measure virtual value.

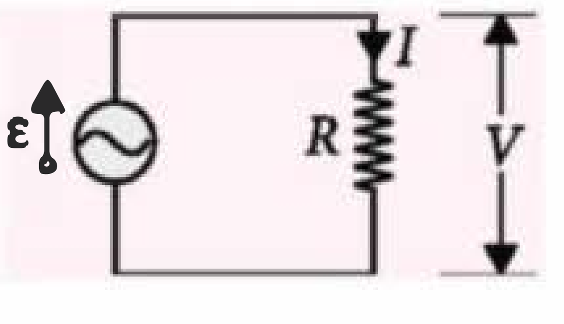
$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = 0.707\, I_0 \text{ or } V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = 0.707\, V_0$$

$$\text{Form factor} = \frac{I_{\text{rms}}}{I_{\text{av}}} = \frac{0.707 I_0}{0.637 I_0} = 1.11$$

AC Voltage Applied to a Resistor

The varying potential difference

$V = V_0 \sin\omega t$



then $I = \frac{V}{R} = \frac{V_0 \sin\omega t}{R} = I_0 \sin \omega t$

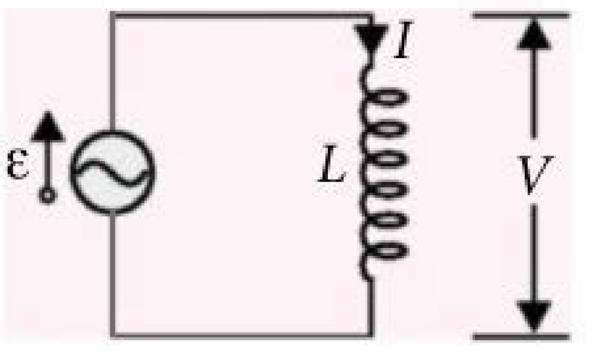
Here the alternating voltage is in phase with current, when ac flows through the resistor.

AC Voltage Applied to an Inductor

Potential difference across the inductor,

$V = V_0 \sin\omega t$

or $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$



where $I_0 = \frac{V_0}{\omega L}$

The alternating current lags behind the alternating voltage by a phase angle of $\pi/2$ when ac flows through an inductor.

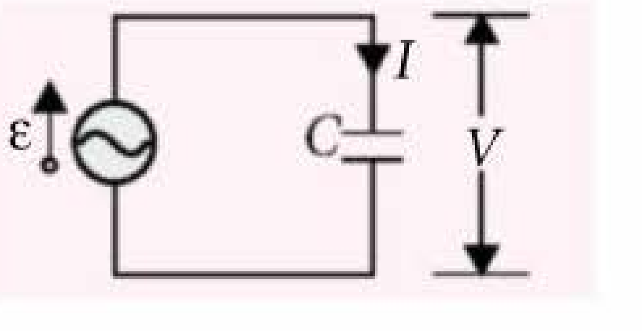
Inductive reactance : It is the opposition offered by the inductor to the flow of alternating current through it. $X_L = \omega L = 2\pi \upsilon L$

AC Voltage Applied to a Capacitor

Potential difference across capacitor,

$V = V_0 \sin\omega t$

or $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$



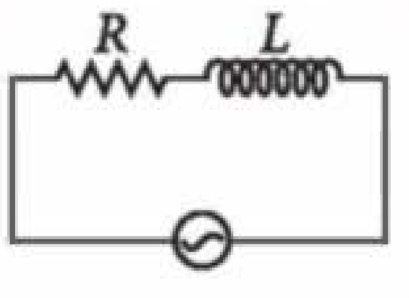
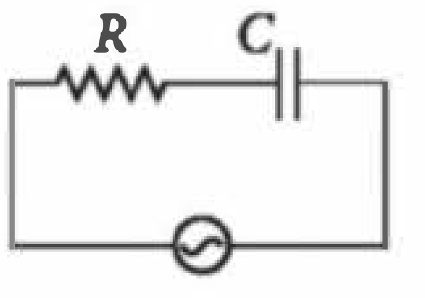
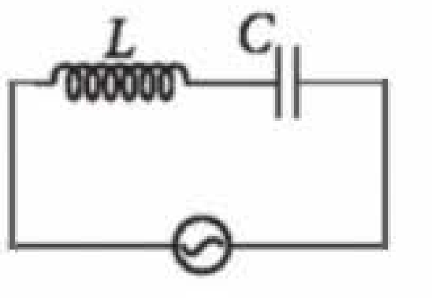
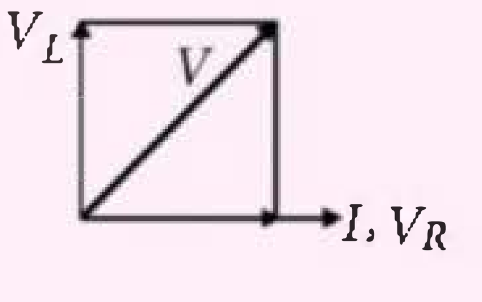
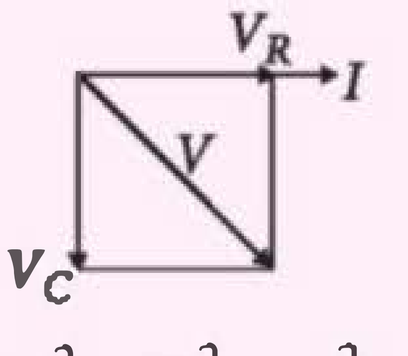
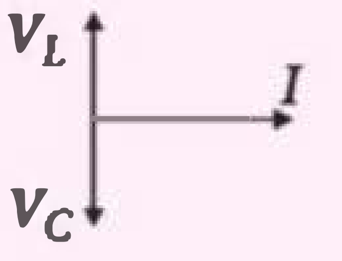
where $I_0 = (\omega C) V_0$

The alternating current leads the voltage by a phase angle of $\pi/2$, when ac flows through a capacitor.

Capacitive reactance : It is the opposition offered by the capacitor to the flow of alternating current through it.

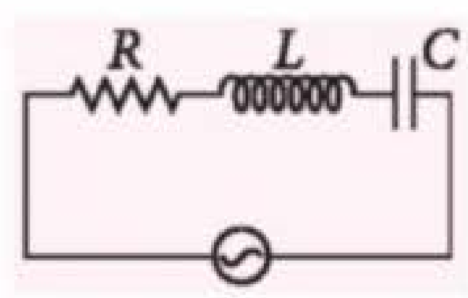
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \upsilon C}$$

Combination of *R – L*, *R – C* and *L – C* in an AC Circuit

Term	*R – L*	*R – C*	*L – C*
Circuit	 I is same in R and L	 I is same in R and C	 I is same in L and C
Phasor diagram	 $V^2 = V_R^2 + V_L^2$	 $V^2 = V_R^2 + V_C^2$	 $V = V_L - V_C$ $(V_L > V_C)$ $V = V_C - V_L$ $(V_C > V_L)$

Phase difference between V and I	V leads I $\left(0 \text{ to } \frac{\pi}{2}\right)$	V lags I $\left(-\frac{\pi}{2} \text{ to } 0\right)$	V lags I $\left(-\frac{\pi}{2}, \text{if } X_C > X_L\right)$ V leads I $\left(+\frac{\pi}{2}, \text{if } X_L > X_C\right)$
Impedance	$Z = \sqrt{R^2 + X_L^2}$	$Z = \sqrt{R^2 + X_C^2}$	$Z = \lvert X_L - X_C \rvert$
At very low υ	$Z \simeq R\ (X_L \to 0)$	$Z \simeq X_C$	$Z \simeq X_C$
At very high υ	$Z \simeq X_L$	$Z \simeq R \quad (X_C \to 0)$	$Z \simeq X_L$

Series *LCR*-Circuit



Let $V = V_0 \sin\omega t$

Then, $I = I_0 \sin(\omega t - \phi)$

where $I_0 = \frac{V_0}{Z}$

Here Z is the impedance of the series LCR circuit.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The alternating current lags behind the voltage by a phase angle ϕ.

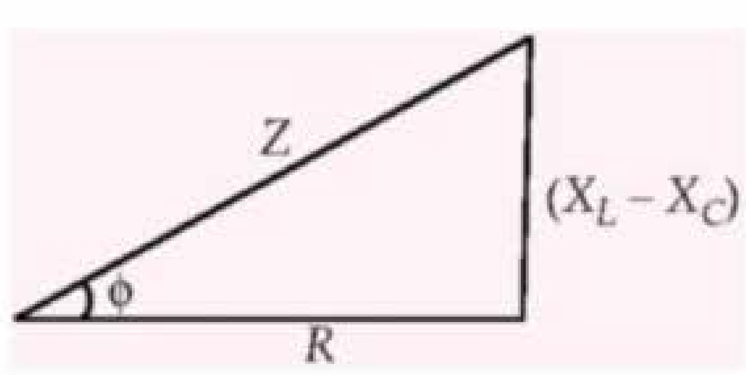
$$\tan\phi = \frac{X_L - X_C}{R}$$

When $X_L > X_C$, $\tan\phi$ is positive. Therefore, ϕ is positive. Hence current lags behind the voltage by a phase angle ϕ. The ac circuit is inductance dominated circuit.

When $X_L < X_C$, $\tan\phi$ is negative. Therefore, ϕ is negative. Hence current leads the voltage by a phase angle ϕ. The ac circuit is capacitance dominated circuit.

Impedance triangle

It is a right angled triangle, whose base represents ohmic resistance (R), perpendicular represents reactance (X_L – X_C) and hypotenuse represents impedance (Z) of the series LCR circuit as shown in figure.



Impedance of circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Admittance

The reciprocal of the impedance of an ac circuit is known as admittance. It is represented by Y.

$\therefore$ Admittance $= \frac{1}{\text{Impedance}}$ or $Y = \frac{1}{Z}$

The unit of admittance is $(\text{ohm})^{-1}$ or siemen.

Susceptance

The reciprocal of the reactance of an ac circuit is known as susceptance. It is represented by S.

$\therefore$ Susceptance $= \frac{1}{\text{Reactance}}$

The unit of susceptance is $(\text{ohm})^{-1}$ or siemen.

- Inductive susceptance $= \frac{1}{\text{Inductive reactance}}$

  or $S_L = \frac{1}{X_L} = \frac{1}{\omega L}$
- Capacitive susceptance $= \frac{1}{\text{Capacitive reactance}}$

  or $S_C = \frac{1}{X_C} = \frac{1}{1/\omega C} = \omega C$

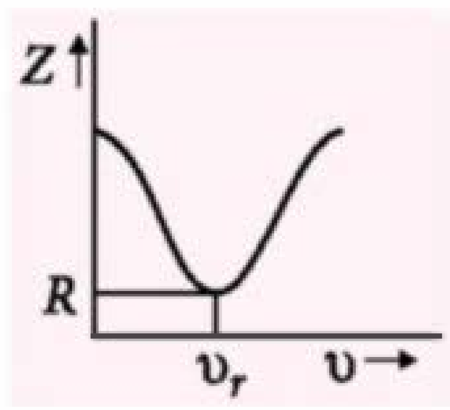
Resonance in Series *LCR* Circuit

- A circuit is said to be resonant when the natural frequency of the circuit is equal to frequency of the applied voltage. For resonance both L and C must be present in circuit.
- At resonance,
  - $X_L = X_C$, $V_L = V_C$
  - $\phi = 0$ (V and I are in same phase)
  - $Z_{min} = R$, $I_{max} = \frac{V}{R}$
- Resonant frequency :

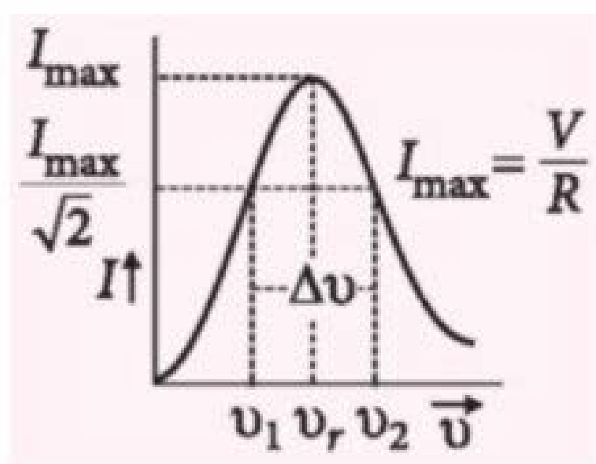
  $\because X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C}$

  $\omega = \frac{1}{\sqrt{LC}}$ or, $\upsilon_r = \frac{1}{2\pi\sqrt{LC}}$
- Variation of Z with υ
  - If $\upsilon < \upsilon_r$ then $X_L < X_C$, circuit is capacitive, (ϕ negative).
  - At $\upsilon = \upsilon_r$, $X_L = X_C$, circuit is resistive, ϕ = zero.
  - If $\upsilon > \upsilon_r$ then $X_L > X_C$ circuit is inductive, (ϕ positive).

As υ increases, Z first decreases then increases.



- Variation of I with υ



as υ increases, I first increases then decreases.

- At resonance impedance of the series resonant circuit is minimum so it is called acceptor circuit as it most readily accepts that current out of many currents whose frequency is equal to its natural frequency. In radio or TV tuning we receive the desired station by making the frequency of the circuit equal to that of the desired station.
- **Bandwidth** $\Delta υ = υ_2 - υ_1$
- **Quality factor (Q) :** Q-factor of ac circuit basically gives an idea about stored energy and lost energy.

$$Q = 2\pi \frac{\text{Maximum energy stored per cycle}}{\text{Maximum energy lost per cycle}}$$

  - It represents the sharpness of resonance.
  - It is unitless and dimensionless quantity.
  - $Q = \frac{(X_L)_r}{R} = \frac{(X_C)_r}{R} = \frac{2\pi υ_r L}{R}$

$$= \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{υ_r}{\Delta υ} = \frac{υ_r}{\text{bandwidth}}$$

  - Sharpness ∝ Quality factor
    R decrease ⇒ Q increases ⇒ sharpness increases

Power in AC Circuit

- Let $V = V_0 \sin \omega t$ and $I = I_0 \sin (\omega t - \phi)$
  Instantaneous power $P = V_0 \sin \omega t \cdot I_0 \sin (\omega t - \phi)$
  $= V_0 I_0 \sin \omega t (\sin \omega t \cos \phi - \sin\phi \cos \omega t)$
- Average power <P>

$$= \frac{1}{T}\int_0^T (V_0 I_0 \sin^2 \omega t \cos \phi - V_0 I_0 \sin \omega t \cos \omega t \sin \phi) dt$$

$$\langle P \rangle = \frac{V_0 I_0 \cos\phi}{2} \Rightarrow \langle P \rangle = V_{rms} I_{rms} \cos\phi$$

- rms power, $P_{rms} = V_{rms} I_{rms}$
- Power factor $(\cos \phi) = \frac{\text{Average power}}{\text{rms power}}$

  $\cos \phi = \frac{R}{Z}$
- Power dissipation is maximum in resistive circuit or at resonance in a LCR series circuit.
- No power is dissipated in purely inductive or capacitive circuit even a current is flowing in the circuit $\left(\because \phi = \frac{\pi}{2}\right)$. This current is referred as wattless current.
- Power is dissipated only in resistor even circuit has RL, RC or LCR combination.

Choke Coil

- Circuit with a choke coil is a series $L - R$ circuit. If resistance of choke coil $= r$ (very small). Current in the circuit, $I = \frac{V}{Z}$ with $Z = \sqrt{(R+r)^2 + (\omega L)^2}$
- It has a high inductance and negligible resistance coil.
- It is used to control current in ac circuit at negligible power loss.

  $\therefore \cos\phi = \frac{V}{Z} = \frac{r}{\sqrt{r^2 + \omega^2 L^2}} \approx \frac{r}{\omega L} \to 0$
- Resistance of an ideal coil is zero.

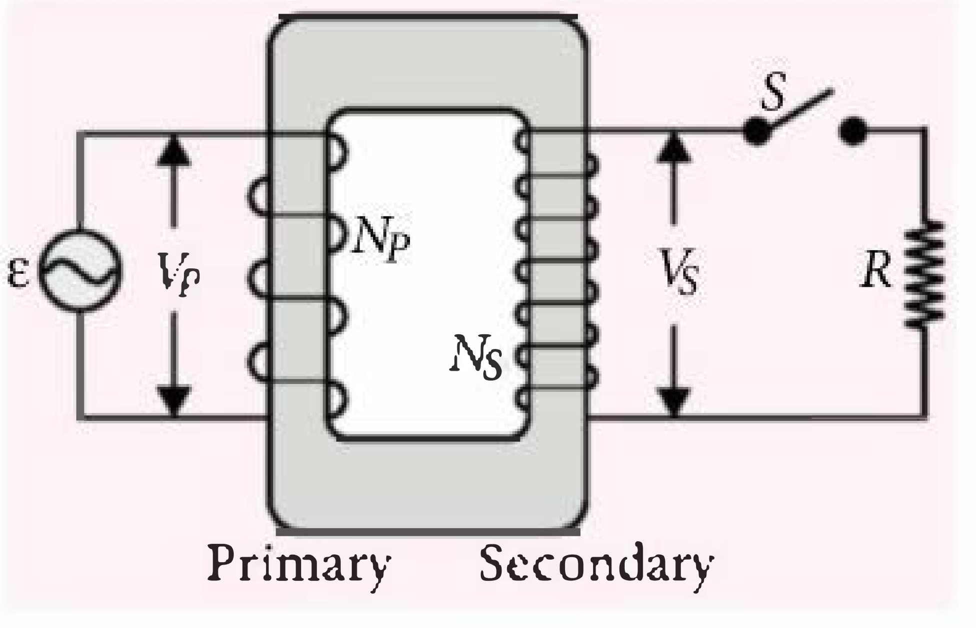
LC Oscillation

- The oscillation of energy between capacitor (electric field energy) and inductor (magnetic field energy) is called LC-oscillation.
- Frequency of oscillation $υ = \frac{1}{2\pi\sqrt{LC}}$
- If charge varies sinusoidally with time t as $q = q_0 \cos \omega t$, then current varies periodically with time t as $I = \frac{dq}{dt} = q_0 \omega \cos\left(\omega t + \frac{\pi}{2}\right)$
- If initial charge on the capacitor is q_0 then electrical energy stored in capacitor is $U_E = \frac{1}{2}\frac{q_0^2}{C}$
- If the capacitor is fully discharged, then total electrical energy is stored in the inductor in the form of magnetic energy.

  $U_B = \frac{1}{2} L I_0^2$, where I_0 = Maximum current

Transformer

- A transformer is an electrical device which is used for changing alternating voltages. It is based on the phenomenon of mutual induction.



- If it is assumed that there is no loss of energy in the transformer, then the power input = the power output, and since $P = I \times V$ then $I_P V_P = I_S V_S$
- Although some energy is always lost, this is a good approximation, since a well designed transformer may have an efficiency of more than 95%.
  $\frac{I_P}{I_S} = \frac{V_S}{V_P} = \frac{N_S}{N_P}$
- A transformer affects the voltage and current. We have
  $V_S = \left(\frac{N_S}{N_P}\right) V_P$ and $I_S = \left(\frac{N_P}{N_S}\right) I_P$
- Now, if $N_S > N_P$, the voltage is stepped up ($V_S > V_P$). This type of arrangement is called a step-up transformer.
- If $N_S < N_P$, we have a step-down transformer. In this case $V_S < V_P$ and $I_S > I_P$. The voltage is stepped down and the current is increased.

Peep Into Previous Years

6. An alternating voltage $v(t) = 220 \sin 100\pi t$ V is applied to a purely resistive load of 50 Ω. The time taken for the current to rise from half of the peak value to the peak value is
(a) 3.3 ms (b) 5 ms (c) 2.2 ms (d) 7.2 ms
(JEE Main 2019)

7. An inductor 20 mH, a capacitor 100 μF and a resistor 50 Ω are connected in series across a source of emf, $V = 10 \sin 314\, t$. The power loss in the circuit is
(a) 0.79 W (b) 0.43 W
(c) 2.74 W (d) 1.13 W ***(NEET 2018)***

Points For Extra Scoring

- For a cylindrical region (of radius R) having time varying magnetic field (dB/dt),

$$\text{induced electric field, } E_{in} = \begin{cases} \frac{r}{2}\left(\frac{dB}{dt}\right); & \text{for } r < R \\ \frac{R^2}{2r}\left(\frac{dB}{dt}\right); & \text{for } r \geq R \end{cases}$$

- The choice of whether the description of an oscillatory motion is by means of sine or cosines or by their linear combinations is unimportant, since changing the zero-time position transforms the one to the other.
- The co-efficient of coupling k between two coils of inductance L_1 and L_2 having mutual inductance M is given by, $k = \frac{M}{\sqrt{L_1 L_2}}$
  The value of k varies from 0 to 1.

Answer Key For Peep Into Previous Years

1. (c)	2. (a)	3. (a)	4. (c)	5. (d)	6. (a)	7. (a)

Website containing database of teachers, academicians launched

The exhaustive database, aimed at aiding teachers' outreach and engagements beyond their institutional geographies, is an initiative of Deepak kumar Mukadam, the chancellor's nominee in the University of Mumbai's management council.

A web portal containing a database of leading professors and academicians across the country has been launched to serve as a resource centre for educational institutions to take better policy decisions. The exhaustive database, aimed at aiding teachers' outreach and engagements beyond their institutional geographies, is an initiative of Deepak kumar Mukadam, the chancellor's nominee in the University of Mumbai's management council.

Maharashtra Higher and Technical Education Minister Vinod Tawde launched the database portal 'academisthan.com' here on 28^{th} August. "This will open up a new window of resources for educational institutes. It is a welcome move of the University of Mumbai," Tawde said.

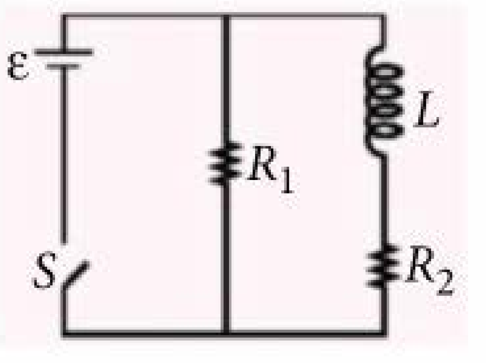
Mukadam said the portal will bring all the top academicians and professors across the country under one roof. "Their work is now just a click away. The portal will serve as a resource centre for educational institutions in government, semi-government and private sectors to develop their policies and other activities," he added. The database will help in providing requisite information to government agencies, NGOs and others to aid in making policy decisions, perspective plans and allocation of resources related to higher education, as per the website. "Academisthan can engage with the government and NGOs for the cause of higher education, academics and benefit of the teachers," it said.

WRAP it up!

1. If $V = 100\sin(100t)$ V and $I = 100\sin\left(100t + \frac{\pi}{3}\right)$ mA are the instantaneous values of voltage and current, then the rms values of voltage and current are respectively
(a) 70.7 V, 70.7 mA (b) 70.7 V, 70.7 A
(c) 141.4 V, 141.4 mA (d) 100 V, 100 mA

2. An express train takes 16 hours to cover the distance of 960 km between Patna and Ghaziabad. The rails are separated by 130 cm and the vertical component of the earth's magnetic field is 4.0×10^{-5} T. If the leakage resistance between the rails is 100 Ω, the retarding force on the train due to the magnetic field will be
(a) 5×10^{-10} N (b) 8×10^{-10} N
(c) 15×10^{-5} N (d) 5×10^{-5} N

3. An inductor of inductance L = 400 mH and resistors of resistances $R_1 = 2\,\Omega$ and $R_2 = 2\,\Omega$ are connected to a battery of emf 12 V as shown in figure.



The internal resistance of the battery is negligible. The switch S is closed at t = 0. The potential drop across L as a function of time is
(a) $6e^{-5t}$ V (b) $12\,e^{-3t}$ V
(c) $6(1 - e^{-t/0.2})$ V (d) $12e^{-5t}$ V

4. An air-core solenoid of length 30 cm, area of cross-section 25 cm^2 and number of turns 500, carries a current of 2.5 A. The current is switched off suddenly in a brief time 10^{-3} s. How much is the average back emf induced across the ends of the open switch, neglecting the end effects?
(a) 2.7 V (b) 9.0 V (c) 3.4 V (d) 6.5 V

5. When an alternating voltage of 220 V is applied across a device X, a current of 0.5 A flows through the circuit and is in phase with the applied voltage. When the same voltage is applied across another device Y, the same current again flows through the circuit, but it lags behind the applied voltage by $\pi/2$ radian. Calculate the current flowing in the circuit when same voltage is applied across the series combination of X and Y.
(a) 0.51 A (b) 0.35 A
(c) 2.50 A (d) 4.57 A

6. A conducting ring of radius 1 m is placed in a uniform magnetic field B of magnitude 0.01 T oscillating with frequency 100 Hz with its plane at right angles to B. What will be the induced electric field?
(a) π V m^{-1} (b) 0.5 V m^{-1}
(c) 1.0 V m^{-1} (d) 6.2 V m^{-1}

7. A uniform magnetic field B exists in a direction perpendicular to the plane of a square frame made of copper wire. The wire has a diameter of 2 mm and a total length of 40 cm. The magnetic field changes with time at a steady rate $dB/dt = 0.02$ T s^{-1}. What will be the current induced in the frame? (Resistivity of copper = $1.7 \times 10^{-8}\ \Omega$ m)
(a) 0.1 A (b) 0.2 A (c) 0.3 A (d) 0.4 A

8. In a uniform magnetic field of induction B, a wire in the form of a semicircle of radius r rotates about the diameter of the circle with angular frequency ω. The axis of rotation is perpendicular to the field. If the total resistance of the circuit is R, then mean power generated per period of rotation is
(a) $\frac{B\pi r^2\omega}{2R}$ (b) $\frac{(B\pi r^2\omega)^2}{8R}$
(c) $\frac{(B\pi r\omega)^2}{2R}$ (d) $\frac{(B\pi r\omega^2)^2}{8R}$

9. If a resistance of 100 Ω, an inductance of 0.5 H and a capacitance of 10×10^{-6} F are connected in series through 50 Hz ac supply, the impedance will be
(a) 1.87 Ω (b) 101.3 Ω
(c) 18.7 Ω (d) 189.7 Ω

10. The instantaneous values of alternating current and voltage in a circuit are given as $I = \frac{1}{\sqrt{2}}\sin(100\pi t)$ A and $\varepsilon = \frac{1}{\sqrt{2}}\sin(100\pi t + \pi/3)$ V
The average power in watts consumed in the circuit is
(a) 1/4 (b) $\sqrt{3}/4$ (c) 1/2 (d) 1/8

11. The flux linked with a coil at any instant t is given by $\phi = 10t^2 - 50t + 250$. Then induced emf at t = 3 s is
(a) −10 V (b) 10 V (c) 190 V (d) −190 V

12. In a series resonant LCR circuit, the voltage across R is 100 V and R = 1 kΩ with C = 2 μF. The resonant frequency ω is 200 rad s^{-1}. At resonance, the voltage across L is
(a) 4×10^{-3} V (b) 2.5×10^{-2} V
(c) 40 V (d) 250 V

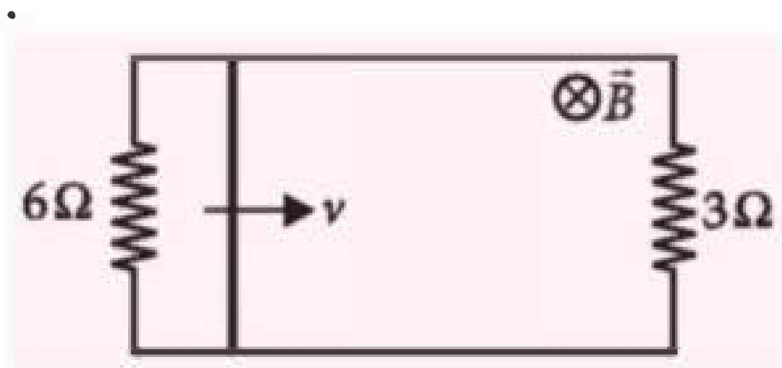
13. An inductor 20 mH, a capacitor 50 μF and a resistor 40 Ω are connected in series across a source of emf $V = 10 \sin 340t$. The power loss in ac circuit is
(a) 0.76 W (b) 0.89 W (c) 0.46 W (d) 0.67 W

14. A long solenoid has 1000 turns. When a current of 4 A flows through it, the magnetic flux linked with each turn of the solenoid is 4×10^{-3} Wb. The self inductance of the solenoid is
(a) 2 H (b) 1 H (c) 4 H (d) 3 H

15. A conducting metal circular-wire-loop of radius r is placed perpendicular to a magnetic field which varies with time as $B = B_0 e^{-\frac{t}{\tau}}$, where B_0 and τ are constants, at time $t = 0$. If the resistance of the loop is R, then the heat generated in the loop after a long time ($t \to \infty$) is
(a) $\dfrac{\pi^2 r^4 B_0^4}{2\tau R}$ (b) $\dfrac{\pi^2 r^4 B_0^2}{2\tau R}$
(c) $\dfrac{\pi^2 r^4 B_0^2 R}{\tau}$ (d) $\dfrac{\pi^2 r^4 B_0^2}{\tau R}$

16. An alternating voltage $V = 200\sqrt{2}\sin(100t)$V is connected to a 1 μF capacitor through an ac ammeter. The reading of ammeter is
(a) 10 mA (b) 20 mA (c) 40 mA (d) 80 mA

17. A rectangular loop with a sliding connector of length l = 1.0 m is situated in a uniform magnetic field B = 2 T, perpendicular to the plane of the loop. Resistance of connector is r = 2 Ω. Two resistances of 6 Ω and 3 Ω are connected as shown in figure. Calculate the external force (in N) required to keep the connector moving with a constant velocity $v = 2$ m s^{-1}.



18. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz ac supply, what value of series inductance (in mH) is required for its function?

19. When a dc voltage of 200 V is applied to a coil of self inductance $2\sqrt{3}/\pi$ H, a current of 1 A flows through it. But by replacing dc source with ac source of 200 V, the current in the coil is reduced to 0.5 A. Find the frequency (in Hz) of ac supply.

20. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the smaller loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is $x \times 10^{-12}$ Wb. Find the value of x.

SOLUTIONS

1. **(a) :** The instantaneous value of voltage is
$V = 100\sin(100t)$ V
Compare it with $V = V_0\sin(\omega t)$ V
we get V_0 = 100 V, ω = 100 rad s^{-1}
The rms value of voltage is
$$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{100}{\sqrt{2}}\text{ V} = 70.7\text{ V}$$
The instantaneous value of current is
$$I = 100\sin\left(100t + \frac{\pi}{3}\right)\text{mA}$$
Compare it with $I = I_0\sin(\omega t + \phi)$
we get I_0 = 100 mA, ω = 100 rad s^{-1}
The rms value of current is
$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{100}{\sqrt{2}}\text{ mA} = 70.7\text{ mA}$$

2. **(a) :** As the train moves in a magnetic field, a motional emf $\varepsilon = Bvl$ is produced across its width. Here, B is the component of the magnetic field in a direction perpendicular to the plane of the motion, *i.e.*, the vertical component.
The speed of the train is, $v = \dfrac{960\text{ km}}{16\text{ h}} = 16.67$ m s^{-1}
Thus, $\varepsilon = (16.67\text{ m s}^{-1})(4.0 \times 10^{-5}\text{ T})(1.3\text{ m})$
$= 8.6 \times 10^{-4}$ V
The leakage current is $I = \varepsilon/R$ and the retarding force is
$$F = IlB = \frac{8.6\times10^{-4}\text{V}}{100\ \Omega}\times1.3\text{ m}\times4.0\times10^{-5}\text{ T}$$
$= 4.47 \times 10^{-10}\text{ N} \simeq 5\times10^{-10}$ N

3. **(d):** If I_1 is the current through R_1 and I_2 is the current through L and R_2, then $I_1 = \varepsilon/R_1$ and $I_2 = I_0(1 - e^{-t/\tau})$, where $\tau = \dfrac{L}{R_2} = \dfrac{400\times10^{-3}}{2} - 0.2$ s
and $I_0 = \dfrac{\varepsilon}{R_2} = \dfrac{12}{2} = 6$ A

Thus, $I_2 = 6(1 - e^{-t/0.2})$

Potential drop across L,

$\varepsilon - R_2 I_2 = 12\ V - 2 \times 6(1 - e^{-t/0.2})\ V = (12e^{-5t})\ V$

4. (d): Given, $l = 30$ cm $= 30 \times 10^{-2}$ m,

Total number of turns $N = 500$

$A = 25\ cm^2 = 25 \times 10^{-4}\ m^2$

$I_1 = 2.5$ A, $I_2 = 0$, $dt = 10^{-3}$ s

$$\text{Induced emf} = \frac{-d\phi_B}{dt} = \frac{d(BAN)}{dt}$$

$$\text{But } B = \mu_0 \frac{N}{l} I \Rightarrow \frac{dB}{dt} = \frac{(\mu_0 N)}{l} \frac{(I_2 - I_1)}{dt}$$

Induced emf

$$= -4\pi \times 10^{-7} \times \frac{500}{30 \times 10^{-2}} \times 25 \times 10^{-4} \times 500 \times \frac{(0-2.5)}{10^{-3}}$$

Induced emf $= 6.5$ V

This is the back emf induced because of cutting off the current.

5. (b): The current and voltage are in phase with each other, when alternating voltage is applied across a resistor. Hence, the device X is a resistor. Its resistance is given by, $R = \frac{220}{0.5} = 440\ \Omega$

The current lags behind the voltage by phase angle $\pi/2$, when the alternating voltage is applied across an inductor. Hence, the device Y is an inductor. Its reactance is given by, $X_L = \frac{220}{0.5} = \frac{220}{0.5} = 440\ \Omega$

Here, $V_{rms} = 220$ V; $R = 440\ \Omega$; $X_L = 440\ \Omega$

If Z is impedance of L-R circuit, then

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{440^2 + 440^2} = 440\sqrt{2}\ \Omega$$

Therefore, current in the L-R circuit,

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{440\sqrt{2}} = 0.3535\ A$$

6. (b): Here, $r = 1$ m, $B = 0.01$ T, $t = \frac{1}{\upsilon} = \frac{1}{100}$ s

$$\text{Induced emf, } |\varepsilon| = \frac{d\phi}{dt}$$

$$|\varepsilon| = \frac{BA}{t} = \frac{B\pi r^2}{t} = \frac{0.01 \times \pi(1)^2}{1/100} = \pi\ V$$

Induced electric field,

$$E = \frac{|\varepsilon|}{l} = \frac{\varepsilon}{2\pi r} = \frac{\pi}{2\pi \times 1} = 0.5\ V\ m^{-1}$$

7. (a): Here, total length $l = 40$ cm $= 40 \times 10^{-2}$ m,

Resistivity $= 1.7 \times 10^{-8}\ \Omega$ m

The area A of the square frame

$$= \left(\frac{40\ cm}{4}\right)\left(\frac{40\ cm}{4}\right) = 0.01\ m^2$$

If the magnetic field at an instant is B, the flux through the frame at that instant will be $\phi = BA$. As the area remains constant, the magnitude of the emf induced will be

$$\varepsilon = \frac{d\phi}{dt} = A\frac{dB}{dt} = (0.01\ m^2)(0.02\ T\ s^{-1}) = 2 \times 10^{-4}\ V$$

The resistance of the frame,

$$R = \frac{(1.7 \times 10^{-8}\ \Omega\ m)(40 \times 10^{-2}\ m)}{3.14 \times 1 \times 10^{-6}\ m^2} = 2.16 \times 10^{-3}\ \Omega$$

Hence, the current induced in the frame will be

$$I = \frac{\varepsilon}{R} = \frac{2 \times 10^{-4}\ V}{2.16 \times 10^{-3}\ \Omega} = 9.3 \times 10^{-2}\ A \approx 0.1\ A$$

8. (b): As $\phi_B = BA\cos\omega t = B\left(\frac{1}{2}\pi r^2\right)\cos\omega t$,

$$\varepsilon = \frac{d\phi_B}{dt} = \frac{1}{2}B\pi r^2\omega\sin\omega t$$

$$\text{Instantaneous power, } P = \frac{\varepsilon^2}{R} = \frac{(B\pi r^2\omega)^2}{4R}\sin^2\omega t$$

$$P_{av} = \frac{\int_0^T P dt}{T} = \frac{(B\pi r^2\omega)^2}{4R}\frac{(T/2)}{T} \quad \left(\because \int_0^T \sin^2\omega t = T/2\right)$$

$$= \frac{(B\pi r^2\omega)^2}{8R}$$

9. (d)

10. (d): As $\varepsilon_{rms} = \frac{\varepsilon_0}{\sqrt{2}} = \frac{(1/\sqrt{2})}{\sqrt{2}} = \frac{1}{2}$ V,

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{(1/\sqrt{2})}{\sqrt{2}} = \frac{1}{2}\ A,$$

and $\cos\phi = \cos\pi/3 = 1/2$

$$P_{av} = \varepsilon_{rms} I_{rms}\cos\phi = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{8}\ W$$

11. (a)

12. (d): Current, $I = V/Z$ ...(i)

where, $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

and $Z = \sqrt{R^2 + (X_L - X_C)^2}$

At resonance, $X_L = X_C$ $\therefore$ $Z = R$

and $V_L = V_C$ $\therefore$ $V = V_R$

$$\therefore \quad I = \frac{V_R}{R} = \frac{100}{1 \times 10^3} = 0.1\ A \quad \text{(Using (i)}$$

Voltage across inductance is V_L

$$\therefore \quad V_L = V_C = IX_C = \frac{I}{\omega C} = \frac{0.1}{200 \times 2 \times 10^{-6}} = 250\ V$$

13. (c): Here, $L = 20$ mH $= 20 \times 10^{-3}$ H,

$C = 50\ \mu F = 50 \times 10^{-6}$ F, $R = 40\ \Omega$,

Given, $V = 10\sin 340t$

Comparing with $V = V_0\sin\omega t$

$\omega = 340$ rad s^{-1} and $V_0 = 10$ V

Now, $X_L = \omega L = 340 \times 20 \times 10^{-3} = 6.8\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{340 \times 50 \times 10^{-6}} = \frac{10^4}{34 \times 5} = 58.82\ \Omega$$

$$Z=\sqrt{R^2+(X_C-X_L)^2}=\sqrt{(40)^2+(58.82-6.8)^2}$$
$$=\sqrt{(40)^2+(52.02)^2}=65.62\ \Omega$$
The peak current in the circuit is
$$I_0=\frac{V_0}{Z}=\frac{10}{65.62}\text{ A},\ \cos\phi=\frac{R}{Z}=\left(\frac{40}{65.62}\right)$$
Power loss in ac circuit,
$$=V_{rms}\,I_{rms}\cos\phi=\frac{1}{2}V_0I_0\cos\phi$$
$$=\frac{1}{2}\times10\times\frac{10}{65.62}\times\frac{40}{65.62}=0.46\text{ W}$$

14. (b)

15. (b): Here, $B=B_0e^{-\frac{t}{\tau}}$

Area of the circular loop, $A=\pi r^2$

Flux linked with the loop at any time t,
$$\phi=BA=\pi r^2B_0e^{-\frac{t}{\tau}}$$
Emf induced in the loop, $\varepsilon=-\frac{d\phi}{dt}=\pi r^2B_0\frac{1}{\tau}e^{-\frac{t}{\tau}}$

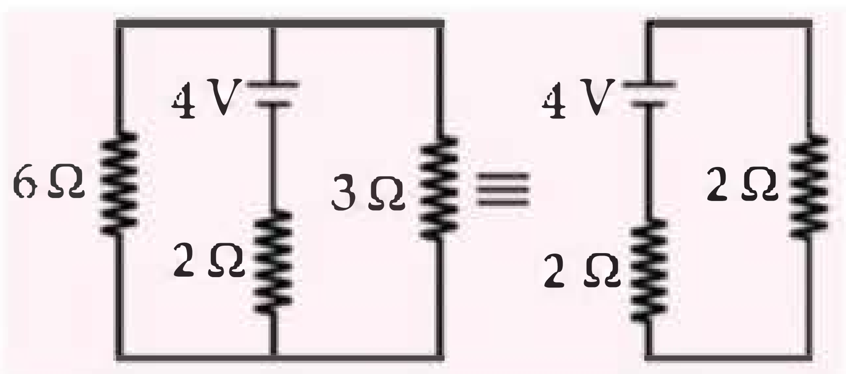
Net heat generated in the loop
$$=\int_0^\infty\frac{\varepsilon^2}{R}dt=\frac{\pi^2r^4B_0^2}{\tau^2R}\int_0^\infty e^{-\frac{2t}{\tau}}dt=\frac{\pi^2r^4B_0^2}{\tau^2R}\times\frac{1}{\left(-\frac{2}{\tau}\right)}\times\left[e^{-\frac{2t}{\tau}}\right]_0^\infty$$
$$=\frac{-\pi^2r^4B_0^2}{2\tau^2R}\times\tau(0-1)=\frac{\pi^2r^4B_0^2}{2\tau R}$$

16. (b): Comparing $V=200\sqrt{2}\sin(100t)$ with $V=V_0\sin\omega t$, we get $V_0=200\sqrt{2}$ and $\omega=100$
$$X_C=\frac{1}{\omega C}=\frac{1}{100\times1\times10^{-6}}=10^4\ \Omega$$
As ac instruments read rms value, the reading of ammeter is
$$I_{rms}=\frac{V_{rms}}{X_C}=\frac{V_0}{\sqrt{2}X_C}\qquad\left(\because V_{rms}=\frac{V_0}{\sqrt{2}}\right)$$
$$=\frac{200\sqrt{2}}{\sqrt{2}\times10^4}=2\times10^{-2}\text{ A}=20\text{ mA}$$

17. (2): Motional emf induced in the connector

$\varepsilon=Blv=2(1)(2)=4$ V

The connector will acts as a cell of emf 4 V and internal resistance 2 Ω. 6 Ω and 3 Ω resistors are in parallel.
$$\therefore\ \frac{1}{R_p}=\frac{1}{6}+\frac{1}{3}$$
$$=\frac{1+2}{6}=\frac{3}{6}=\frac{1}{2}$$

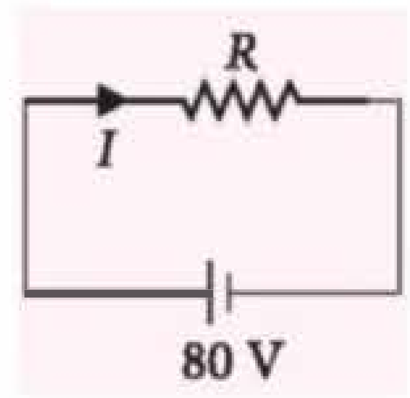


$R_p=2\ \Omega$

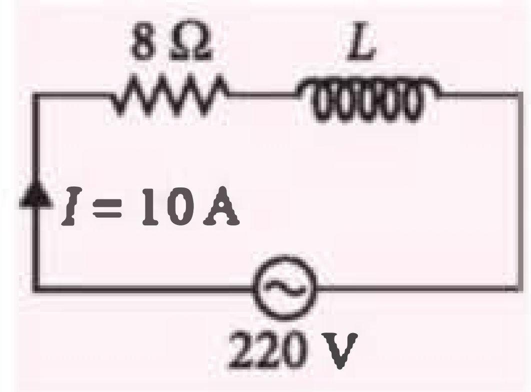
$\therefore$ Current through the connector,
$$=\frac{\varepsilon}{R_p+r}=\frac{4}{2+2}=1\text{ A}$$
Magnetic force on the connector $=BIl=2(1)(1)=2$ N

18. (65) : For a dc source

$I=10$ A, $V=80$ V



Resistance of the arc lamp,
$$R=\frac{V}{I}=\frac{80}{10}=8\ \Omega$$
For an ac source, $\varepsilon_{rms}=220$ V



$\upsilon=50$ Hz

$\omega=2\pi\times50=100\,\pi$ rad s^{-1}

Arc lamp will glow if $I=10$ A,
$$\therefore\ I=\frac{\varepsilon_{rms}}{\sqrt{R^2+\omega^2L^2}}$$
or $R^2+\omega^2L^2=\left(\frac{\varepsilon_{rms}}{I}\right)^2$ or $8^2+(100\,\pi)^2L^2=\left(\frac{220}{10}\right)^2$

or $L^2=\frac{22^2-8^2}{(100\,\pi)^2}\quad\therefore\ L=\frac{\sqrt{420}}{100\,\pi}=0.065$ H

19. (50) : For dc voltage source, $R=\frac{V}{I}=\frac{200}{1}=200\ \Omega$

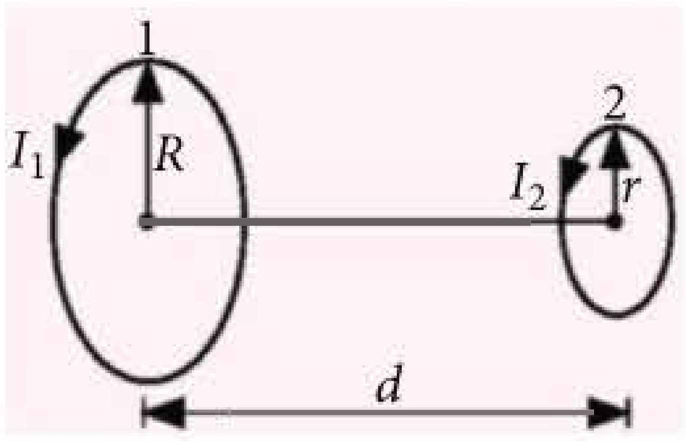
and $L=\frac{2\sqrt{3}}{\pi}$ H

When dc source is replaced by ac source,

Impedance, $Z=\frac{E_v}{I_v}=\frac{200}{0.5}=400\ \Omega$

Hence the reactance of inductor
$$X_L=\sqrt{Z^2-R^2}\qquad[\because Z^2=R^2+X_L^{\ 2}]$$
$$=\sqrt{400^2-200^2}=200\sqrt{4-1}=200\sqrt{3}$$
Now, $X_L=\omega L=2\pi\upsilon L=200\sqrt{3}$
$$\upsilon=\frac{200\sqrt{3}}{2\pi L}=\frac{200\sqrt{3}}{2\pi\times2\sqrt{3}/\pi}=50\text{ Hz}$$

20. (91) : As field due to current loop 1 at an axial point



$$\therefore\ B_1=\frac{\mu_0I_1R^2}{2(d^2+R^2)^{3/2}}$$
Flux linked with smaller loop 2 due to B_1 is
$$\phi_2=B_1A_2=\frac{\mu_0I_1R^2}{2(d^2+R^2)^{3/2}}\pi r^2$$
The coefficient of mutual inductance between the loops is $M=\frac{\phi_2}{I_1}=\frac{\mu_0R^2\pi r^2}{2(d^2+R^2)^{3/2}}$

Flux linked with bigger loop 1 is
$$\phi_1=MI_2=\frac{\mu_0R^2\pi r^2I_2}{2(d^2+R^2)^{3/2}}$$
Substituting the given values, we get
$$\phi_1=\frac{4\pi\times10^{-7}\times(20\times10^{-2})^2\times\pi\times(0.3\times10^{-2})^2\times2}{2[(15\times10^{-2})^2+(20\times10^{-2})^2]^{3/2}}$$
$\phi_1=9.1\times10^{-11}$ Wb.

◆◆

CBSE warm-up!

CLASS-XII

Chapterwise Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2019-20.

Series 5 — Electromagnetic Waves | Optics

Time Allowed : 3 hours
Maximum Marks : 70

GENERAL INSTRUCTIONS

(i) All questions are compulsory.

(ii) Section A : Q. no. 1 to 20 are very short answer-objective questions and carry 1 mark each.

(iii) Section B : Q. no. 21 to 27 are short answer questions and carry 2 marks each.

(iv) Section C : Q. no. 28 to 34 are long answer-I questions and carry 3 marks each.

(v) Section D : Q. no. 35 to 37 are long answer-II questions and carry 5 marks each.

(vi) There is no overall choice in the question paper. However, internal choices are given in the sections.

(vii) Use log tables if necessary, use of calculators is not allowed.

SECTION-A

1. A short pulse of white light is incident from air to a glass slab at normal incidence. After travelling through the slab, the first colour to emerge is
   (a) blue (b) green
   (c) violet (d) red

2. To observe diffraction, the size of the obstacle
   (a) should be $\lambda/2$, where λ is the wavelength
   (b) should be of the order of wavelength
   (c) has no relation to wavelength
   (d) should be much larger than the wavelength.

3. Displacement current goes through the gap between the plates of a capacitor when the charge on the capacitor
   (a) is changing with time (b) decreases
   (c) decreases to zero (d) all of the above.

4. A. Wavelength of microwaves is greater than that of ultraviolet rays.
   B. The wavelength of infrared rays is lesser than that of ultraviolet rays.
   C. The wavelength of microwaves is lesser than that of infrared rays.
   D. Gamma ray has shortest wavelength in the electromagnetic spectrum.
   Choose the correct option.
   (a) A and B are true (b) B and C are true
   (c) C and D are true (d) A and D are true

5. The phenomena involved in the reflection of radiowaves by ionosphere is similar to
   (a) reflection of light by a plane mirror
   (b) total internal reflection of light
   (c) dispersion of light by water molecules during the formation of a rainbow
   (d) scattering of light by the particles of air.

6. For light diverging from a point source
   (a) the wavefront is spherical

(b) the wavefront is cylindrical
(c) the wavefront is parabolic
(d) the intensity at the wavefront does not depend on the distance.

7. A microwave and an ultrasonic sound wave have the same wavelength. Their frequencies are in the ratio (approximately)
(a) 10^2 (b) 10^4 (c) 10^6 (d) 10^8

8. Which of the following electromagnetic waves is used in medicine to destroy cancer cells?
(a) IR-rays (b) Visible rays
(c) Gamma rays (d) Ultraviolet rays

9. The final image in an astronomical telescope with respect to object is
(a) virtual and erect (b) real and erect
(c) real and inverted (d) virtual and inverted.

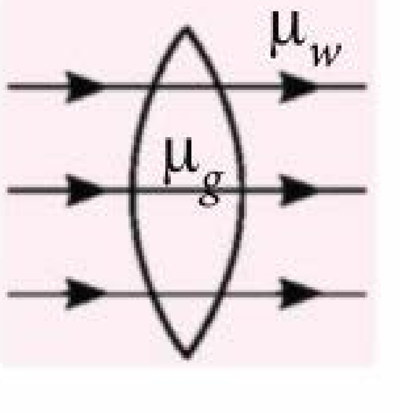
10. From Brewster's law, except for polished metallic surfaces, the polarising angle
(a) depends on wavelength and is different for different colours
(b) independent of wavelength and is different for different colours
(c) independent of wavelength and is same for different colours
(d) depends on wavelength and is same for different colours.

11. A variable frequency ac source is connected to a capacitor. How will the displacement current change with decrease in frequency?

12. Give the ratio for velocities of light rays of wavelength 4000 Å and 8000 Å in vacuum.

13. Why does a ray of light bend towards normal as it passes from air to glass ?

14. In the figure shown, path of a parallel beam of light passing through a convex lens of refractive index μ_g kept in a medium of refractive index μ_w is shown. Is (i) $\mu_g = \mu_w$ or (ii) $\mu_g > \mu_w$ or (iii) $\mu_g < \mu_w$?

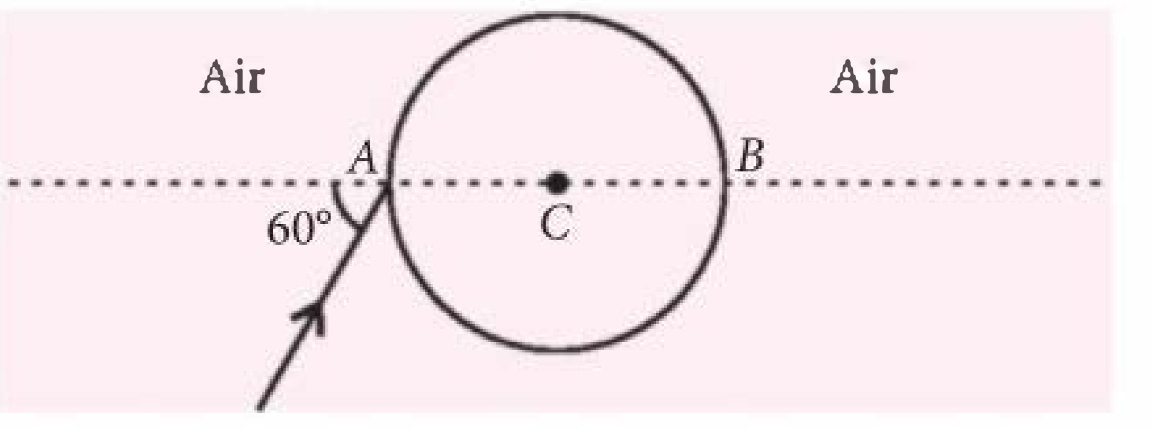


15. Which of the following waves can be polarized (a) Heat waves (b) Sound waves? Give reason to support your answer.

16. Why is the resolving power of a microscope having oil immersion objective high?

17. For the same value of angle of incidence, the angles of refraction in three media A, B and C are 15°, 25° and 35° respectively. In which media would the velocity of light be minimum?

18. A ray of light falls on a transparent sphere with centre C as shown in the figure. The ray emerges from the sphere parallel to the line AB. Find the angle of refraction at A if refractive index of the material of the sphere is $\sqrt{3}$.



19. For a given single slit, the diffraction pattern is obtained on a fixed screen, first by using red light and then with blue light. In which case, will the central maxima, in the observed diffraction pattern, have a larger angular width?

20. Which part of electromagnetic spectrum has largest penetrating power?

SECTION-B

21. A plane electromagnetic wave travels, in vacuum, along the y-direction. Write (a) the ratio of the magnitudes, and (b) directions of its electric and magnetic field vectors.

22. The human eye has an approximate angular resolution of $\phi = 5.8 \times 10^{-4}$ rad and a typical photoprinter prints a minimum of 300 dpi (dots per inch, 1 inch = 2.54 cm). At what minimal distance z should a printed page be held so that one does not see the individual dots.

23. (a) A person standing before a concave mirror cannot see his inverted image unless he stands beyond the centre of curvature. Why?
(b) Using mirror formula, explain why does a convex mirror always produce a virtual image.

OR

Explain with reason, how the power of a diverging lens changes when (a) it is kept in a medium of refractive index greater than that of the lens. (b) incident red light is replaced by violet light.

24. For a glass prism ($\mu = \sqrt{3}$) the angle of minimum deviation is equal to the angle of the prism. Find the angle of the prism.

25. Why is no interference pattern observed when two coherent sources are
(a) infinitely close to each other?
(b) far apart from each other?

OR

The critical angle between a given transparent medium and air is denoted by i_c. A ray of light travelling through air enters this transparent medium at an angle of incidence equal to the polarizing angle (i_p). Deduce a relation for the angle of refraction (r_p) in terms of i_c.

26. (a) An electromagnetic wave is travelling in a medium with a velocity $\vec{v} = v\hat{i}$. Draw a sketch showing the propagation of the electromagnetic wave, indicating the direction of the oscillating electric and magnetic fields.

(b) Identify the electromagnetic waves whose wavelengths vary as

(i) 10^{-12} m < λ < 10^{-8} m

(ii) 10^{-3} m < λ < 10^{-1} m

Write one use for each.

27. How are X-rays produced? Write their two important uses.

SECTION - C

28. Define the term, "refractive index" of a medium. Verify Snell's law of refraction when a plane wavefront is propagating from a denser to a rarer medium.

29. Answer the following questions:

(a) Name the electromagnetic waves which are suitable for radar systems used in aircraft navigation. Write the range of frequency of these waves.

(b) If the Earth did not have atmosphere, would its average surface temperature be higher or lower than what it is now? Explain.

(c) An electromagnetic wave exerts pressure on the surface on which it is incident. Justify.

OR

Give four basic properties of electromagnetic waves.

30. An object of size 3.0 cm is placed 14 cm in front of a concave lens of focal length 21 cm.

(a) What is the nature, size and position of the image formed?

(b) What happens if the object is moved further away from the lens?

31. (a) (i) What is meant by a linearly polarized light?

(ii) Describe briefly using a diagram how sunlight is polarised?

(b) Unpolarised light is incident on a polaroid. How would the intensity of transmitted light change when the polaroid is rotated?

32. (a) A mobile phone lies along the principal axis of a concave mirror. Show, with the help of a suitable diagram, the formation of its image. Explain why magnification is not uniform.

(b) Suppose the lower half of the concave mirror's reflecting surface is covered with an opaque material. What effect will this have on the image of the object? Explain.

OR

(a) Draw a ray diagram showing the path of a ray of light entering through a triangular glass prism.

(b) Deduce the expression for the refractive index of glass prism in terms of the angle of minimum deviation and angle of the prism.

33. Two towers on top of two hills are 40 km apart. The line joining them passes 50 m above a hill halfway between the towers. What is the longest wavelength of radiowaves, which can be sent between the towers without appreciable diffraction effects?

34. Two convex lenses, of equal focal length, but of aperture A_1 and A_2(A_2 < A_1) are used as the objective lenses in two astronomical telescopes having identical eyepieces. Compare the ratio of their (a) resolving power (b) (normal) magnifying power and (c) intensity of images formed by them. Which one of the two telescope should be preferred? Why?

SECTION - D

35. Draw a ray diagram to show the image formation by a concave mirror when the object is kept between its focus and the pole. Using this diagram, derive the magnification formula for the image formed.

OR

Draw a ray diagram showing the formation of the image by a point object on the principal axis of a spherical convex surface separating two media of refractive indices n_1 and n_2, when a point source is kept in rarer medium of refractive index n_1. Derive the relation between object and image distance in terms of refractive index of the medium and radius of curvature of the surface. Hence obtain the expression for lens-maker's formula in the case of thin convex lens.

36. (a) Distinguish between linearly polarised and unpolarised light.

(b) Show that the light waves are transverse in nature.

(c) Why does light from a clear blue portion of the sky show a rise and fall of intensity when viewed through a polaroid which is rotated? Explain by drawing the necessary diagram.

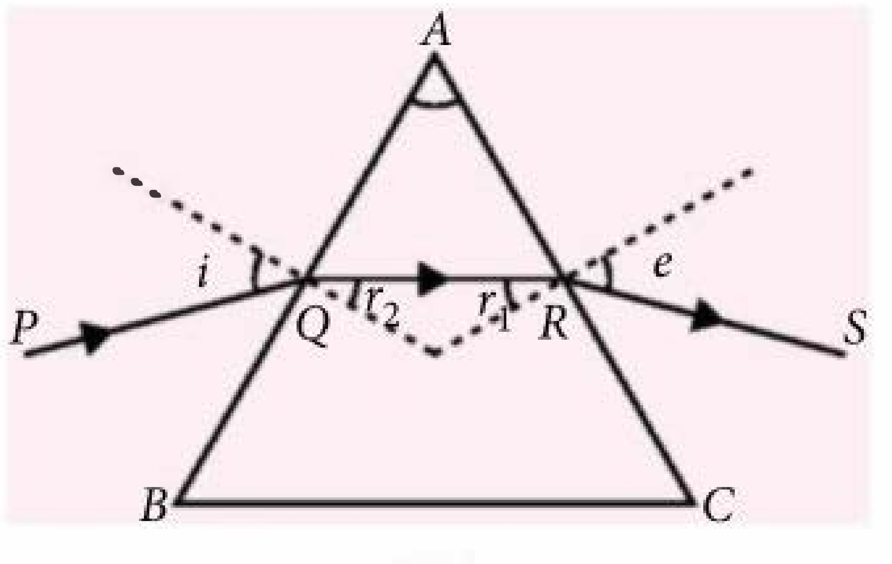
OR

(a) In Young's double slit experiment, derive the condition for
   (i) constructive interference and
   (ii) destructive interference at a point on the screen.

(b) A beam of light consisting of two wavelengths, 800 nm and 600 nm is used to obtain the interference fringes in a Young's double slit experiment on a screen placed 1.4 m away. If the two slits are separated by 0.28 mm, calculate the least distance from the central bright maximum where the bright fringes of the two wavelengths coincide.

37. (a) A ray of light passing from air through an equilateral glass prism undergoes minimum deviation when the angle of incidence is 3/4 of the angle of prism. Calculate the speed of light in the prism.

(b) Figure shows a ray of light passing through a prism. If the refracted ray QR is parallel to the base BC, show that (i) $r_1 = r_2 = A/2$, (ii) angle of minimum deviation, $D_m = 2i - A$.



OR

(a) Draw a ray diagram showing image formation in a compound microscope. Define the term limit of resolution and name the factors on which it depends. How is it related to resolving power of a microscope?

(b) Suggest two ways by which the resolving power of a microscope can be increased.

(c) A telescope resolves whereas a microscope magnifies. Justify this statement.

SOLUTIONS

1.	(d)	2.	(b)	3.	(d)	4.	(d)
5.	(b)	6.	(a)	7.	(c)	8.	(c)
9.	(d)	10.	(a)				

11. With the decrease in frequency, the reactance, $X_C = \frac{1}{2\pi\upsilon C}$ increases.
This decreases conduction current. As $I_D = I_C$, the displacement current will also decrease.

12. Ratio = 1, because light rays of both wavelengths travel with the same velocity in vacuum.

13. We know that $\mu = \frac{\sin i}{\sin r} = \frac{c}{v}$

As the speed of light in air is greater than that in glass *i.e.*, $c > v$, so $\sin i > \sin r$ or $\angle i > \angle r$. Hence a ray of light bends towards the normal as it passes from air to glass.

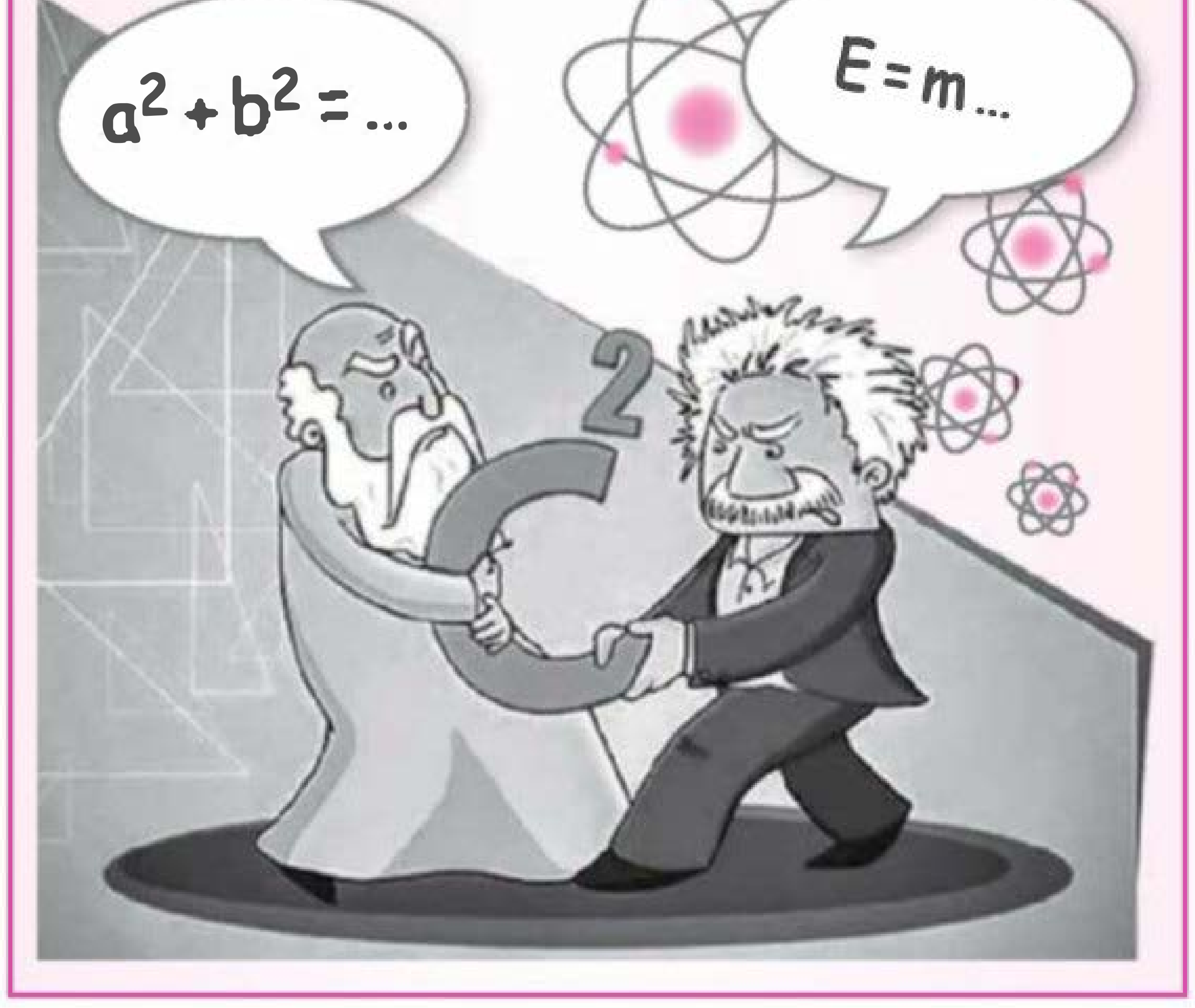
14. $\mu_g = \mu_w$.

15. As only the transverse wave can be polarized, that is why the heat waves which are transverse wave and have vibrations perpendicular to the direction of propagation can be polarized whereas the sound waves cannot be polarized being longitudinal in nature and having vibrations in the direction of propagation.

16. Resolving power of a microscope $= \frac{2\mu \sin\theta}{\lambda}$

Thus such a microscope uses oil of high refractive index (μ) between the object and the objective, in order to have a high resolving power.

COMIC CAPSULE



17. Refractive index, $\mu = \frac{c}{v} = \frac{\sin i}{\sin r}$

As $\sin 15° < \sin 25° < \sin 35°$

So, $v_A < v_B < v_C$

Hence in medium *A*, velocity of light is minimum.

18. From Snell's law, we have : $\frac{\sin(i)}{\sin(r)} = \mu$

At *A*, $i = 60°$; $\mu = \sqrt{3}$

Now, $\sin(r) = \frac{\sin(i)}{\mu}$

$\Rightarrow \sin(r) = \frac{\sin(60°)}{\sqrt{3}} = \frac{1}{2} \Rightarrow r = \sin^{-1}\left(\frac{1}{2}\right)$

$\therefore \quad r = 30°$

19. Angular width of central maxima is given by

$2\theta = \frac{2\lambda}{a}$

Since $\lambda_r > \lambda_b$. Therefore, width of central maxima of red light is greater than the width of central maxima of blue light.

20. Gamma rays (frequency range > 3×10^{21} Hz) has largest penetrating power.

21. **(a)** $E/B = c$, speed of light

(b) For an electromagnetic wave travelling along *y*-direction, its electric and magnetic field vectors are along *z*-axis and *x*-axis respectively. The direction of $\bar{E} \times \bar{B}$ is same as that of direction of wave propagation and $\hat{k} \times \hat{i} = \hat{j}$.

22. Given, angular resolution of human eye,

$\phi = 5.8 \times 10^{-4}$ rad

Linear distance between two successive dots in a typical photoprinter is $l = \frac{2.54}{300} = 0.85 \times 10^{-2}$ cm

We know $\phi = \frac{l}{z}$, $\phi = \frac{0.85 \times 10^{-2}}{z}$

$z = \frac{0.85 \times 10^{-2}}{5.8 \times 10^{-4}} = 14.7$ cm.

23. **(a)** A concave mirror forms real inverted image when the object is placed beyond *F*. When the person stands between *F* and *C*, the image is formed beyond *C* *i.e.*, behind the man and the man is not able to see his image. When he stands beyond *C*, real and inverted image is formed between *F* and *C* *i.e.*, in front of him and so he can see his image.

(b) For convex mirror : $f > 0$, $u < 0$

Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{f} - \frac{1}{(-u)} \Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{u} \Rightarrow v = \frac{f \times u}{f + u}$

$\therefore \quad v > 0$

This implies that image of object placed in front of a convex mirror is always formed behind the mirror which is virtual in nature.

OR

$P = \frac{1}{f} = \left(\frac{\mu_2 - \mu_1}{\mu_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

For a diverging lens : $R_1 = -R$, $R_2 = +R$

$P = \left(\frac{\mu_2 - \mu_1}{\mu_1}\right)\left(-\frac{2}{R}\right)$ = a negative value

(a) When $\mu_1 > \mu_2$, $P = \frac{1}{f}$ = a positive value

Thus, the lens becomes converging.

(b) $(\mu_2)_{violet} > (\mu_2)_{red}$

The power of the lens increases when red light is replaced by violet light.

24. Given, $\mu = \sqrt{3}$, $\delta_m = A$

Angle of minimum deviation,

$\mu = \frac{\sin((A + \delta_m)/2)}{\sin(A/2)}$

$\sqrt{3} = \frac{\sin\left(\frac{A+A}{2}\right)}{\sin(A/2)} = \frac{2\sin(A/2)\cos(A/2)}{\sin(A/2)}$ $(\because \delta_m = A)$

$\sqrt{3} = 2\cos(A/2)$

$\cos(A/2) = \frac{\sqrt{3}}{2}$ or $\frac{A}{2} = 30°$ $\therefore$ $A = 60°$

25. Fringe width, $\beta = \frac{D\lambda}{d}$ *i.e.*, $\beta \propto \frac{1}{d}$

(a) when the two coherent sources are placed infinitely close to each other, the fringe width becomes very large. Even a single fringe may occupy the entire screen. The interference pattern is not observed.

(b) As the distance between the sources is increased, the fringe width goes on decreasing. At very large separation, it becomes too small to be detected. The interference pattern cannot be observed.

OR

According to Brewster's law, when a ray of light is incident on a transparent refracting medium at polarising angle i_p, then $\mu = \tan i_p$

But $i_p + r_p = 90°$ or $i_p = 90° - r_p$

$\therefore \quad \mu = \tan(90° - r_p) = \cot r_p = \frac{1}{\tan r_p}$ ...(i)

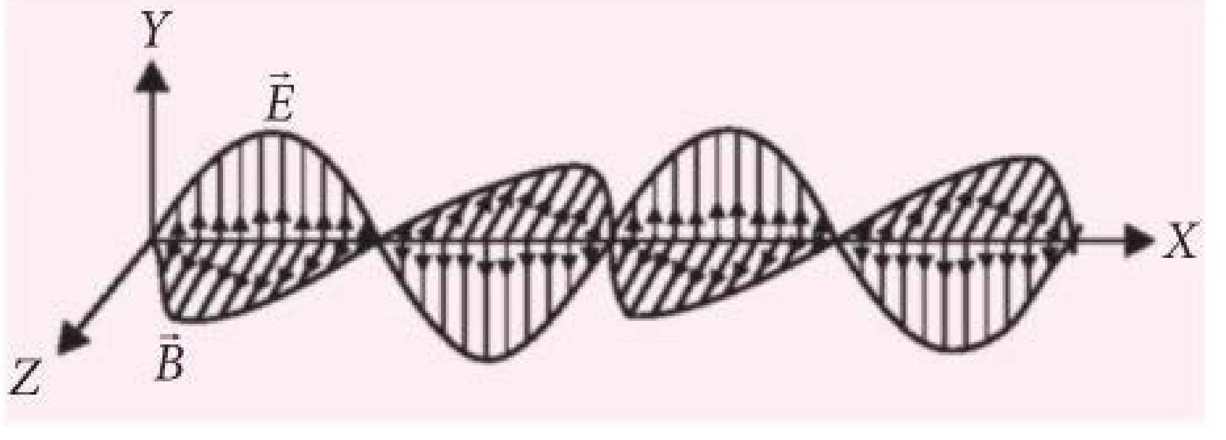
As i_c is the critical angle for the transparent medium,

so, $\mu = \frac{1}{\sin i_c}$ ...(ii)

On comparing (i) and (ii), we get

$\tan r_p = \sin i_c$ or $r_p = \tan^{-1}(\sin i_c)$.

26. (a) In figure shown, the velocity of propagation of electromagnetic wave is along X-axis $\vec{v} = v\hat{i}$ and electric field $\vec{E}$ along Y-axis and magnetic field $\vec{B}$ along Z-axis.



(b) (i) X-rays - used to study atomic structure.

(ii) Microwaves - used in radar application.

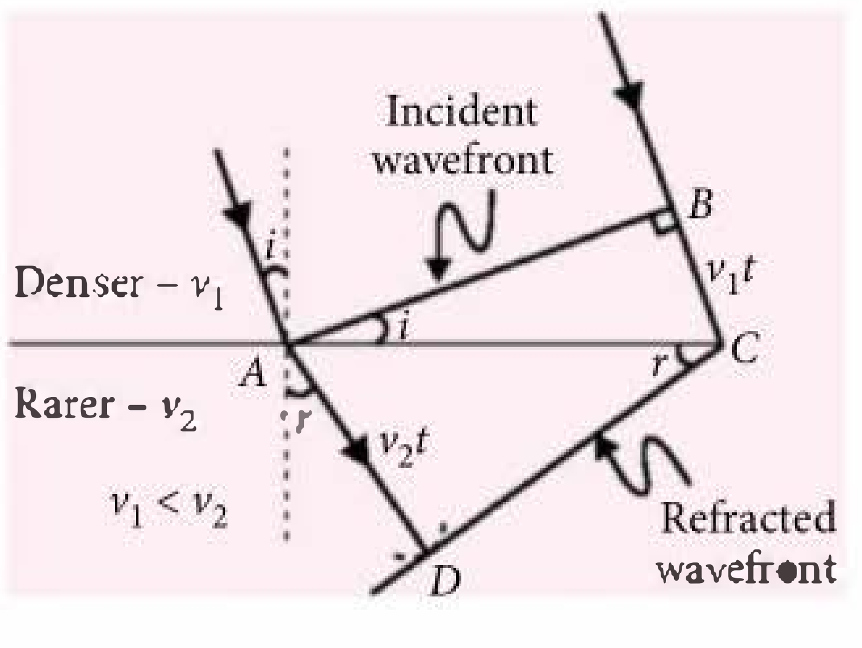
27. Production of X-rays : When high energetic electrons strike a metallic target of high atomic weight and high melting point, X-rays are produced. In production of X-rays mechanical energy of electrons is converted as electromagnetic energy of X-rays.

Uses : (i) X-rays are used in medical diagnostics to detect fractures in bones, tuberculosis of lungs, presence of stone in gall bladder and kidney. (ii) They are used in engineering to check flaws in bridges. In physics X-rays are used to study crystal structure.

28. Refractive index (μ) : Refractive index of a medium is defined as the ratio of the speed of light in vacuum to the speed of light in that medium. *i.e.*,

$$\mu = \frac{c}{v} = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in that medium}}$$

Given figure shows the refraction of a plane wavefront at a rarer medium *i.e.*, $v_2 > v_1$



Let the angles of incidence and refraction be i and r respectively.

From right ΔABC, we have, $\sin \angle BAC = \sin i = \frac{BC}{AC}$

From right ΔADC, we have,

$\sin \angle DCA = \sin r = \frac{AD}{AC} \quad \therefore \quad \frac{\sin i}{\sin r} = \frac{BC}{AD} = \frac{v_1 t}{v_2 t}$

or $\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = {}^1\mu_2$ (a constant)

This verifies Snell's law of refraction. The constant ${}^1\mu_2$ is called the refractive index of the second medium with respect to first medium.

29. (a) Microwaves are suitable for the radar system used in aircraft navigation. Range of frequency of microwaves is 10^8 Hz to 10^{11} Hz.

(b) If the Earth did not have atmosphere, then there would be absence of green house effect due to the atmosphere. Due to this reason, the temperature of the earth would be lower than what it is now.

(c) An electromagnetic wave carries momentum with itself and given by $p = \frac{\text{Energy of wave } (U)}{\text{Speed of the wave } (c)}$

When it is incident upon a surface it exerts pressure on it.

OR

The basic properties of electromagnetic waves are :

(i) Electromagnetic waves are produced by accelerated charges and do not require any medium for their propagation.

(ii) The oscillations of $\vec{E}$ and $\vec{B}$ fields are perpendicular to each other as well as to the direction of propagation of the wave. So, the electromagnetic waves are transverse in nature.

(iii) All electromagnetic waves travel in free space with the same speed, $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8$ m s^{-1}.

(iv) The amplitude ratio of electric and magnetic fields is $\frac{E_0}{B_0} = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

30. (a) Object of size 3 cm is placed 14 cm in front of concave lens.

Lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{-14} = \frac{1}{-21}; \frac{1}{v} = -\frac{1}{21} - \frac{1}{14} = \frac{-2-3}{42}$

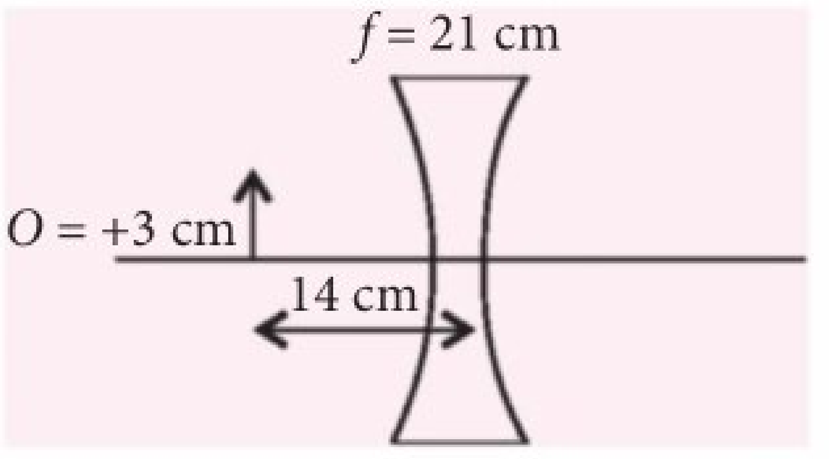
$v = -\frac{42}{5} = -8.4$ cm

Formula for magnification

$$m = \frac{I}{O} = +\frac{v}{u}$$

or $\frac{I}{O} = \frac{v}{u}$

or $\frac{I}{+3} = \frac{-8.4}{-14}$



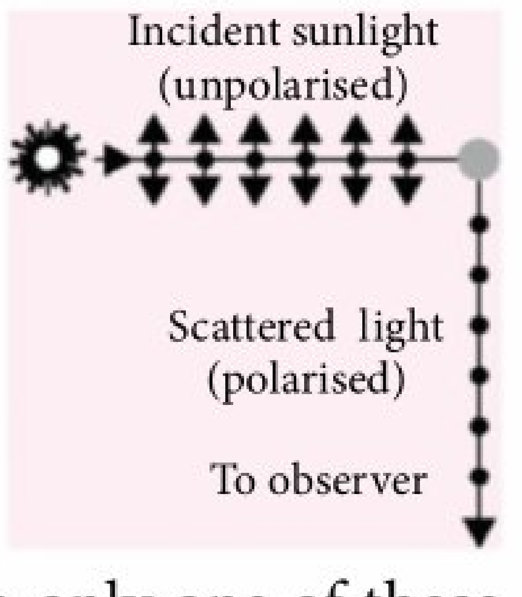
$\Rightarrow \quad I = +1.8$ cm

So, the image is virtual, erect, of the size 1.8 cm and is located 8.4 cm from the lens on the same side as object.

(b) As the object is moved away from the lens, the virtual image moves towards the focus of the lens but never beyond it. The image also reduces in size as it shifts towards focus.

31. (a) (i) If the vibrations of a wave are present only in one direction in a plane perpendicular to the direction of propagation, then these waves is said to be linearly polarised.

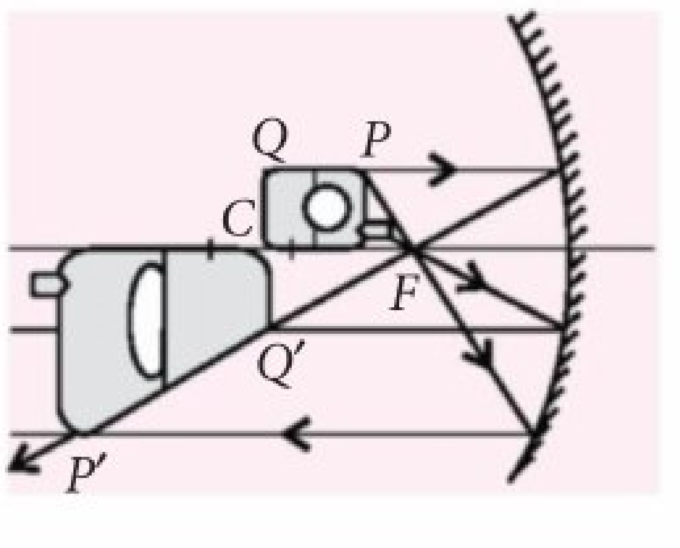
(ii) The acceleration of the charges, in the scattering molecules, due to the electric field of the incident radiation, can be in two mutually perpendicular directions. The observer, however, received the scattered light, corresponding to only one of these two sets of the accelerated charges.



This causes scattered light to get polarised.

(b) When unpolarsied light is incident on a polaroid, then transmitted light will be polarised with half the intensity of unpolarised light. When polaroid is rotated, then the intensity of transmitted light remains constant.

32. (a) The formation of the image of the cell phone is shown in figure. The part which is at C will be imaged at C and will be of the same size, *i.e.*, $Q'C = QC$. The other end P of the mobile phone is highly magnified by the concave mirror.

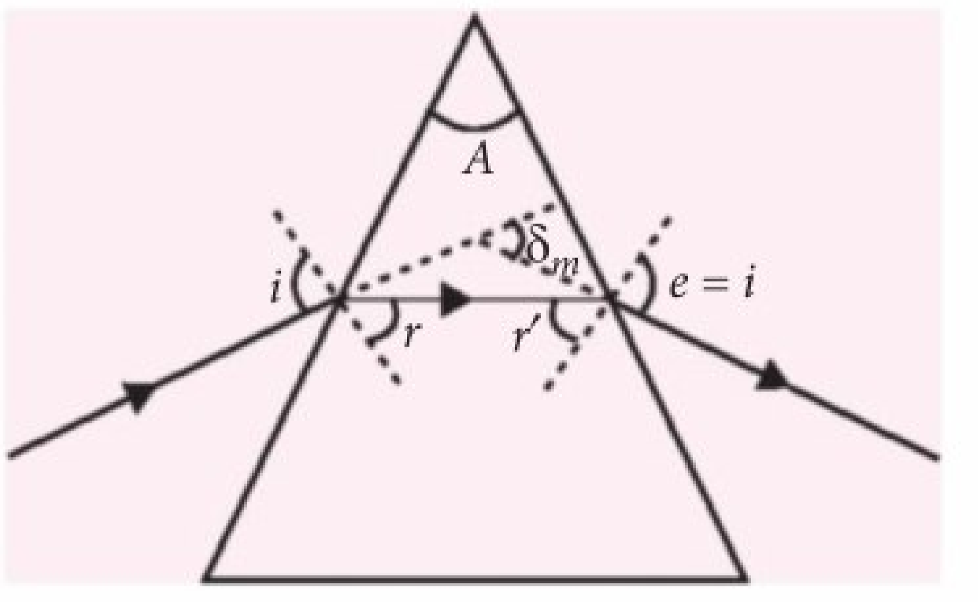


Thus the different parts of the mobile phone are magnified in different proportions because of their different locations from the concave mirror.

(b) One would naturally think that image will be half of the object, but taking the laws of reflection to be true for all points of the mirror, the image will be of the whole object. However, as the area of the reflecting surface has reduced, the intensity of the image will be dim.

OR

(a) The ray diagram showing the path of a way of light entering through a triangular glass prism is given below.



(b) At minimum deviation, the inside beam travels parallel to base of the prism.

Then, $i = e$

$r = r'$

$\delta_m = (i + e) - (r + r')$

$\delta_m = 2i - 2r$ ...(i)

Also $r + r' = A = 2r$ ...(ii)

So, angle of incidence using equation (i)

$i = \frac{A + \delta_m}{2}$, angle of refraction $r = \frac{A}{2}$

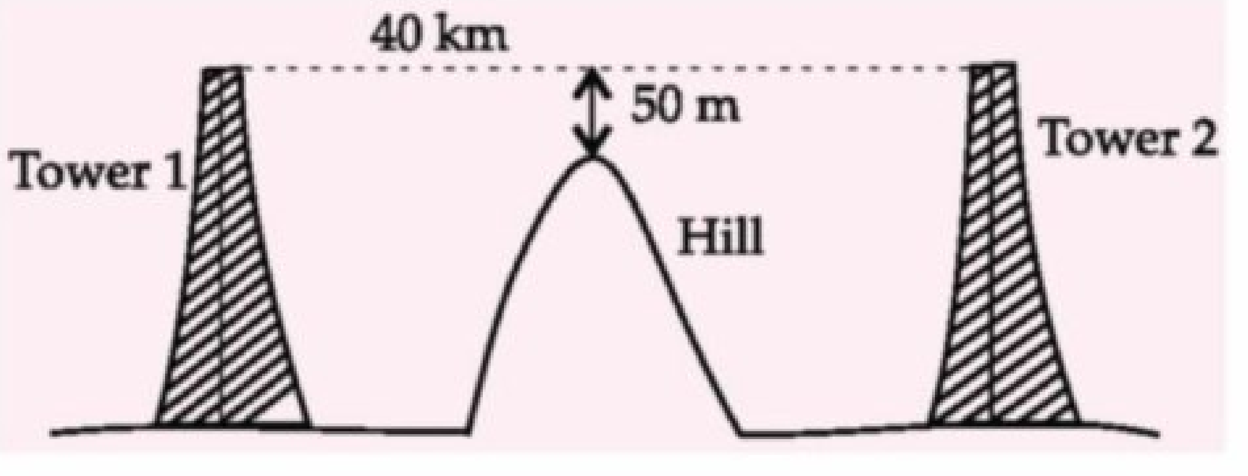
Now refractive index of the material of prism

$$^{a}\mu_g = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$$

where A is the angle of the prism and $A = 60°$ for an equilateral prism.

33. For diffraction of radiowaves not to occur the distance of middle hill from the source should be less than fresnel distance for a slit width 'a' of 50 m.

$$D_F = \frac{a^2}{\lambda}$$



Distance between one of the towers and the hill halfway in between

$$= \frac{40}{2}\text{ km} = 20000\text{ m} = 20\times10^3\text{ m}$$

Longest wavelength of radiowave which can be sent without appreciable diffraction effect

$$\lambda = \frac{a^2}{D_F} = \frac{50\times50}{20\times10^3} = 12.5\text{ cm}$$

Thus wavelength of radio waves longer than 12.5 cm will bend due to the hill in the middle of the towers.

34. (a) Resolving power of a telescope $= \dfrac{D}{1.22\lambda}$

$$\Rightarrow \frac{(R.P.)_1}{(R.P.)_2} = \frac{A_1}{A_2} > 1$$

(b) Magnifying power of a telescope (in normal adjustment) $= f_0/f_e$

$$\therefore \frac{(M.P.)_1}{(M.P.)_2} = 1$$

because f_0 and f_e are same in both cases.

(c) Intensity ratio of images

$$\frac{I_1}{I_2} = \frac{A_1}{A_2} > 1$$

The telescope with objective of aperture A_1 should be preferred for viewing as this would

(i) give a better resolution.

(ii) have a higher light gathering power.

35. *Refer to answer 14, Page no. 215* (MTG CBSE Champion Physics Class 12).

OR

Refer to answer 63, Page no. 224 (MTG CBSE Champion Physics Class 12).

36. *Refer to answer 107, Page no. 270* (MTG CBSE Champion Physics Class 12).

OR

Refer to answer 45, Page no. 259 (MTG CBSE Champion Physics Class 12).

37. (a) *Refer to answer 81, Page no. 226* (MTG CBSE Champion Physics Class 12).

(b) *Refer to answer 71, Page no. 225* (MTG CBSE Champion Physics Class 12).

OR

Refer to answer 123, Page no. 234 (MTG CBSE Champion Physics Class 12).

Education Ministry's 'Shagun' Portal to Connect 92 Lakh Teachers, 26 Crore Students

The Human Resource Development (HRD) Ministry on 28th August launched a school education portal 'Shagun' with an aim to integrate over 2.3 lakh educational websites. The web portal is set to link 15 lakh schools across the country and will provide teachers, students and parents all the information that they need on school education. According to Union HRD minister Ramesh Pokhriyal Nishank the schools have been geo-tagged due to which all the data provided by them will be accessed through 'Shagun'.

The HRD Ministry officials while talking about the security level and accuracy of the information said that there will also be a third-party verification of the information provided by schools and websites. These schools and websites include – 1,200 Kendriya Vidyalayas, 600 Navodaya Vidyalayas, 18,000 CBSE-affiliated schools, 30 SCERTs, 19,000 organisations affiliated with NCTE among others are integrated with the portal.

HRD Ministry officials further said, "Report cards of 15 lakh schools all over the country will be available on the newly created junction. The portal seeks to connect approximately 92 lakh teachers and 26 crore students. The website provides a very robust feedback mechanism. Common people can directly give their feedback about schools which will further increase the public participation and will ensure accountability and transparency." They added, "The single source of information will immensely benefit all the stakeholders, including parents and the general public, heads of schools, teachers, students, the policy makers, the officials and the researchers. The website also provides vital information relating to the availability of nearby schools, navigable distance vis-a vis aerial distance between schools."

The ministry said the Shagun (htpp://shagun.govt.in/) is an over-arching initiative to improve school education system by creating a junction for all online portals and websites.

MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 120

Magnetic Effects of Current and Magnetism

Time Taken : 60 Min.

NEET / AIIMS

Only One Option Correct Type

1. The magnetic potential due to a magnetic dipole at a point on its axis distant 40 cm from its centre is found to be 2.4×10^{-5} J A^{-1} m^{-1}. The magnetic moment of the dipole will be
(a) 28.6 A m^2 (b) 32.2 A m^2
(c) 38.4 A m^2 (d) 22.2 A m^2

2. A circular coil of radius R carries an electric current. The magnetic field due to the coil at a point on the axis of the coil located at a distance r from the centre of the coil such that $r >> R$, varies as
(a) $\frac{1}{r}$ (b) $\frac{1}{r^{1/2}}$ (c) $\frac{1}{r^2}$ (d) $\frac{1}{r^3}$

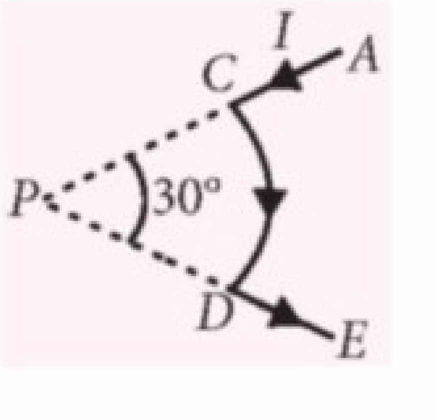
3. A beam of protons with a velocity 4×10^5 m s^{-1} enters a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. Find the pitch of the helix. (Mass of the proton = 1.67×10^{-27} kg)
(a) 2.35 cm (b) 5.35 cm
(c) 4.35 cm (d) 6.35 cm

4. Two long parallel straight conductors carry currents I_1 and I_2 ($I_1 > I_2$). When the currents are in the same direction, the magnetic field at a point midway between the wires is 20 μT. If the direction of I_2 is reversed, the field becomes 50 μT. The ratio of the currents $\frac{I_1}{I_2}$ is
(a) $\frac{5}{2}$ (b) $\frac{7}{3}$ (c) $\frac{4}{3}$ (d) $\frac{2}{3}$

5. Magnetic field at the centre of a circular loop of area A is B. The magnetic moment of the loop is
(a) $\frac{BA^2}{\mu_0 \pi}$ (b) $\frac{BA\sqrt{A}}{\mu_0}$
(c) $\frac{BA\sqrt{A}}{\mu_0 \pi}$ (d) $\frac{2BA\sqrt{A}}{\mu_0 \sqrt{\pi}}$

6. In a vibration magnetometer, the time period of a bar magnet oscillating in horizontal component of earth's magnetic field is 2 s. When a magnet is brought near and parallel to it, the time period reduces to 1 s. The ratio $\frac{F}{H}$ of the fields, F due to magnet and H, the horizontal component will be
(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{3}$ (d) 3

7. A current path shaped as shown in figure produces a magnetic field at P, the centre of the arc. If the arc subtends an angle of 30° and the radius of the arc is 0.6 m. What is the magnitude of the field at P if the current is 3.0 A?



(a) 2.62×10^{-6} T
(b) 3.62×10^{-7} T
(c) 2.62×10^{-7} T
(d) 4.67×10^{-8} T

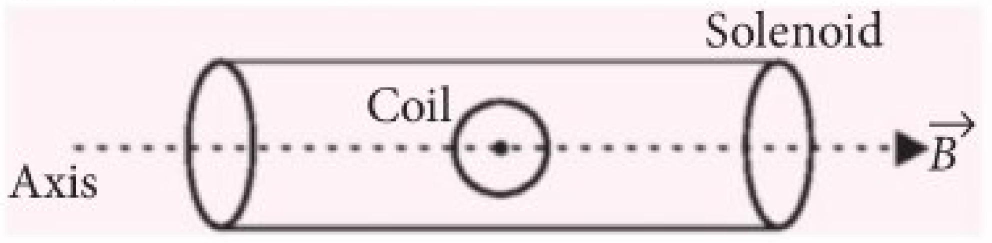
8. A proton is projected with a uniform velocity v along the axis of a current carrying solenoid, then
(a) the proton will be accelerated along the axis.
(b) the proton path will be circular about the axis.
(c) the proton moves along helical path.
(d) the proton will continue to move with velocity v along the axis.

9. A long solenoid has 800 turns per metre length of solenoid. A current of 1.6 A flows through it. The magnetic induction at the end of the solenoid on its axis is
(a) 16×10^{-4} T (b) 8×10^{-4} T
(c) 32×10^{-4} T (d) 4×10^{-4} T

10. Which of the following characteristics is not associated with a ferromagnetic material?
(a) It is strongly attracted by a magnet.
(b) It tends to move from a region of strong magnetic field to a region of weak magnetic field.
(c) Ferromagnetism is produced due to spin of electrons.
(d) Above the Curie temperature, it exhibits paramagnetic properties.

11. A 0.1 m long conductor carrying a current of 50 A is held perpendicular to a magnetic field of 1.25 mT magnitude. The maximum mechanical power required to move the conductor with a speed of 1 m s^{-1} is
(a) 62.5 mW (b) 625 mW
(c) 6.25 mW (d) 12.5 mW

12. The torque required to hold a small circular coil of 10 turns, area 1 mm^2 and carrying a current of $\left(\frac{21}{44}\right)$A in the middle of a long solenoid of 10^3 turns per metre carrying a current of 2.5 A, with its axis perpendicular to the axis of the solenoid is



(a) 1.5×10^{-6} N m (b) 1.5×10^{-8} N m
(c) 1.5×10^{6} N m (d) 1.5×10^{8} N m

Assertion & Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as :
(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

13. **Assertion :** The sensitivity of a moving coil galvanometer is increased by placing a suitable magnetic material as a core inside the coil.
Reason : Soft iron has a high magnetic permeability and cannot be easily magnetized or demagnetized.

14. **Assertion :** The true geographic north direction is found by using a compass needle.
Reason : The magnetic meridian of the earth is along the axis of rotation of the earth.

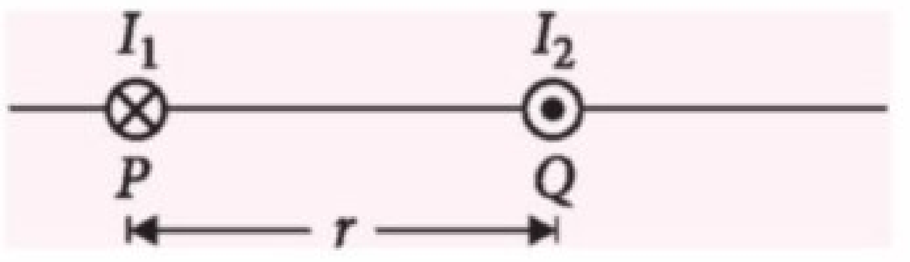
15. **Assertion :** The net force on a closed circular current carrying loop placed in a uniform magnetic field is zero.
Reason : The torque produced in a conducting circular ring is zero when it is placed in a uniform magnetic field such that the magnetic field is perpendicular to the plane of loop.

JEE MAIN / ADVANCED

Only One Option Correct Type

16. At a certain location in Africa, a compass points 12° west of the geographic north. The north tip of the magnetic needle of a dip circle placed in the plane of magnetic meridian points 60° above the horizontal. The horizontal component of the earth's field is measured to be 0.16 G. The direction and magnitude of the earth's field at the location, is
(a) 32° west of geographical meridian and 3.2×10^{-4} T
(b) 12° west of geographical meridian and 0.32×10^{-4} T
(c) 12° east of geographical meridian and 0.32×10^{-4} T
(d) 32° east of geographical meridian and 3.2×10^{-4} T

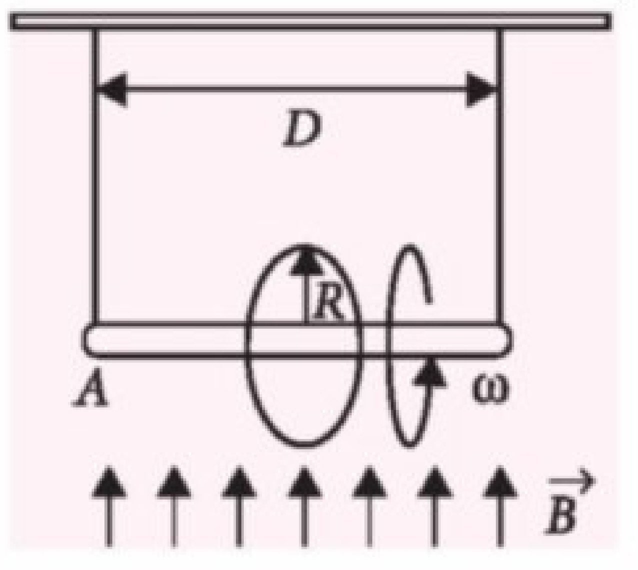
17. Two parallel wires P and Q placed at a separation of r = 6 cm carry electric currents I_1 = 5 A and I_2 = 2 A in opposite directions as shown in figure. Find the point on the line PQ where the resultant magnetic field is zero.



(a) 4 cm to the right of Q
(b) 9 cm to the left of P
(c) 2 cm to the right of P
(d) 3 cm to the left of Q

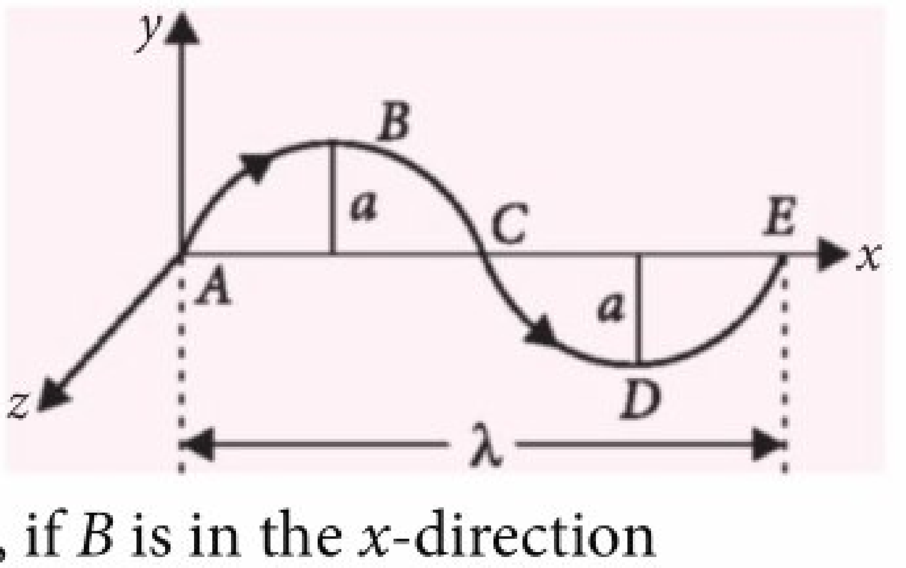
18. A long straight wire of radius a carries a steady current I. The current is uniformly distributed across its cross-section. The ratio of the magnetic field at $\frac{a}{2}$ and $2a$ from its centre is
(a) 1 : 4 (b) 4 : 1 (c) 1 : 1 (d) 1 : 2

19. A ring of radius R having uniformly distributed charge Q is mounted on a rod suspended by two identical strings. The tension in strings in equilibrium is T_0. Now a vertical magnetic field is switched on and the ring is rotated at constant angular velocity ω. Find the maximum ω with which the ring can be rotated if the strings can withstand a maximum tension of $3T_0/2$.



More than One Options Correct Type

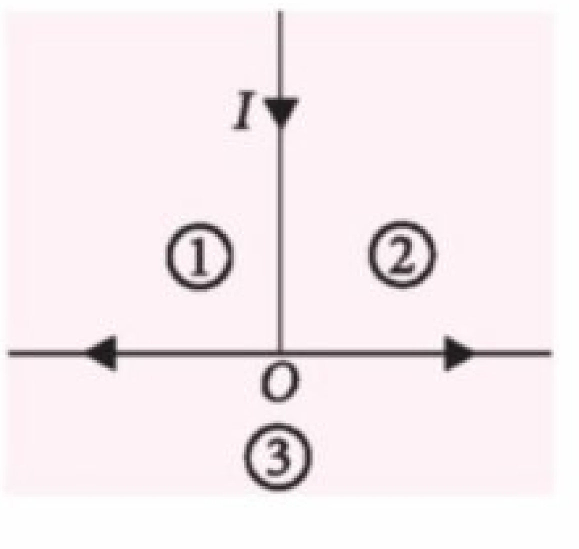
20. A conductor $ABCDE$ as shown, carries a current I. It is placed in the xy plane with the ends A and E on the x-axis. A uniform magnetic field of magnitude B exists in the region. The force acting on it will be



(a) zero, if B is in the x-direction
(b) λBI in the z-direction, if B is in the y-direction
(c) λBI in the negative y-direction, if B is in the z-direction
(d) $2aBI$, if B is in the x-direction.

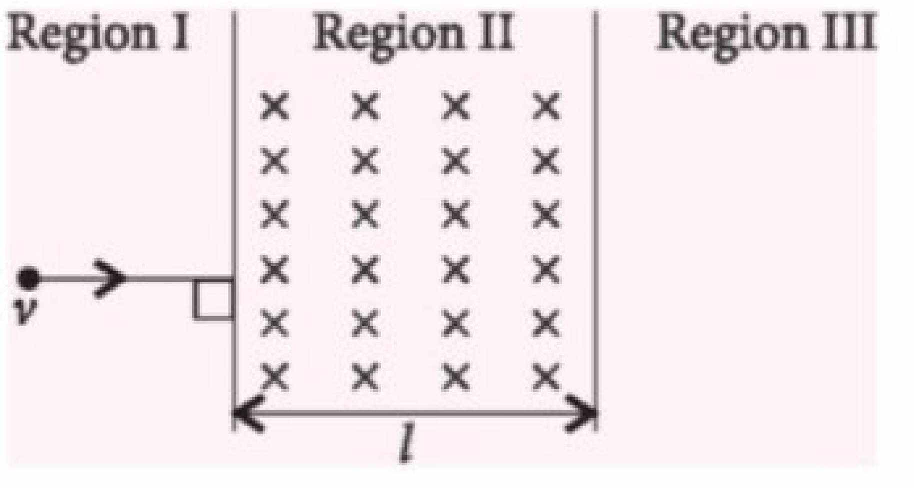
21. A microammeter has a resistance of 100 Ω and a full scale range of 50 μA. It can be used as a voltmeter or as a higher range ammeter provided a resistance is added to it. Pick the correct range and resistance combination(s).
(a) 50 V range with 10 kΩ resistance in series.
(b) 10 V range with 200 kΩ resistance in series.
(c) 5 mA range with 1 Ω resistance in parallel.
(d) 10 mA range with 1 Ω resistance in parallel.

22. A direct current I flows along a long straight wire as shown in the figure. From point O the current spreads radially all over on infinite conducting plane perpendicular to the wire. Then



(a) magnetic field in region ① is non-uniform
(b) magnetic field in region ② is non-uniform
(c) magnetic field in region ③ is non-uniform
(d) all of these.

23. A particle of mass m and charge q moving with velocity v enters region II normal to the boundary as shown in the figure.



Region II has a uniform magnetic field B perpendicular to the plane of the paper. The length of the region II is l. Choose the correct choice(s).
(a) The particle enters region III only if its velocity $v > \frac{qlB}{m}$

(b) The particle enters region III only if its velocity $v < \frac{qlB}{m}$

(c) Path length of the particle in region II is maximum when velocity $v = \frac{qlB}{m}$

(d) Time spent in region II is same for any velocity v as long as the particle returns to region I.

24. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time $t = 0$, this charge has velocity $\vec{v}$ in the x-y plane, making an angle θ with the x-axis. Which of the following option(s) are correct for time $t > 0$?

(a) If $\theta = 0°$, the charge moves in a circular path in the x-z plane.

(b) If $\theta = 0°$, the charge undergoes helical motion with constant pitch along the y-axis.

(c) If $\theta = 10°$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis.

(d) If $\theta = 90°$, the charge undergoes linear but accelerated motion along the y-axis.

25. A charged particle with velocity $\vec{v} = x\hat{i} + y\hat{j}$ moves in a magnetic field $\vec{B} = y\hat{i} + x\hat{j}$. The magnitude of magnetic force acting on the particle is F. Which one of the following statement(s) are correct?

(a) No force will act on particle, if $x = y$.

(b) $F \propto (x^2 - y^2)$ if $x > y$.

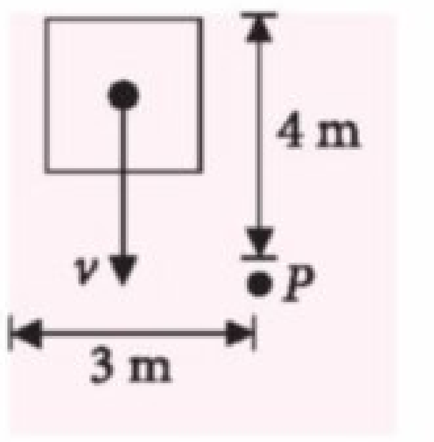
(c) The force will act along z-axis, if $x > y$.

(d) The force will act along y-axis, if $y > x$.

Numerical Value Type

26. A current $I = 10$ A flows in a ring of radius $r_0 = 15$ cm made of a very thin wire. The tensile strength of the wire is equal to $T = 1.5$ N. The ring is placed in a magnetic field, which is perpendicular to the plane of the ring so that the forces tend to break the ring. Find B (in T) at which the ring is broken.

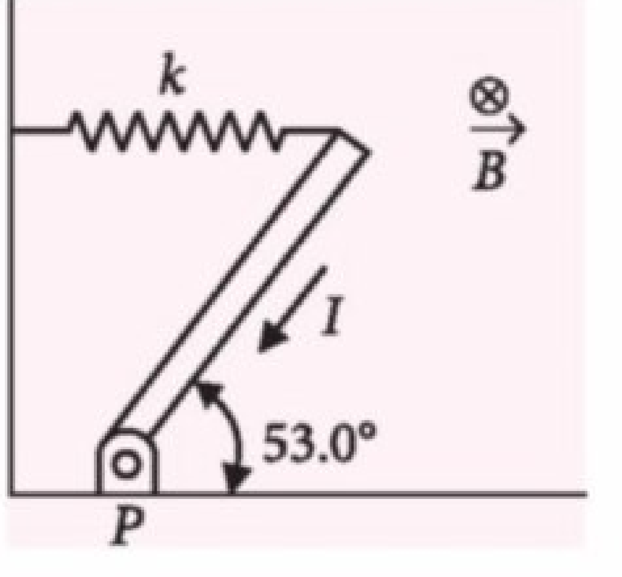
27. An elevator carrying a charge of 0.5 C is moving down with a velocity of 5×10^3 m s^{-1}. The charge particle is 4 m from the bottom and 3 m horizontally from P as shown in figure. What magnetic field (in μT) does it produce at point P?



28. An iron rod of volume 10^{-4} m^3 and relative permeability 1000 is placed inside a long solenoid wound with 5 turns per cm. If a current of 0.1 A is passed through the solenoid, find the magnetic moment (in A m^2) of the rod.

Comprehension Type

A thin, uniform rod with negligible mass and length 0.200 m is attached to the floor by a frictionless hinge at point P as shown in the figure. A horizontal spring with force constant $k = 4.80$ N m^{-1} connects the other end of the rod to a vertical wall.



The rod is in a uniform magnetic field $B = 0.340$ T directed into the plane of the paper. There is a current $I = 6.50$ A in the rod, in the direction shown.

29. Calculate the torque due to the magnetic force on the rod, for an axis at P.

(a) 0.0442 N m^{-1}, clockwise

(b) 0.0442 N m^{-1}, anticlockwise

(c) 0.022 N m^{-1}, clockwise

(d) 0.022 N m^{-1}, anticlockwise

30. When the rod is in equilibrium and makes an angle of 53.0° with the floor, is the spring stretched or compressed?

(a) 0.05765 m, stretched

(b) 0.05765 m, compressed

(c) 0.0242 m, stretched

(d) 0.0242 m, compressed

❖❖

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ......
No. of questions correct ......
Marks scored in percentage ......

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

PHYSICS MUSING

Physics Musing was started in August 2013 issue of Physics For You. The aim of Physics Musing is to augment the chances of bright students preparing for JEE (Main and Advanced) / NEET / AIIMS / JIPMER with additional study material.

In every issue of Physics For You, 10 challenging problems are proposed in various topics of JEE (Main and Advanced) / NEET. The detailed solutions of these problems will be published in next issue of Physics For You.

The readers who have solved five or more problems may send their detailed solutions with their names and complete address. The names of those who send atleast five correct solutions will be published in the next issue.

We hope that our readers will enrich their problem solving skills through "Physics Musing" and stand in better stead while facing the competitive exams.

PROBLEM Set 75

SINGLE OPTION CORRECT TYPE

1. A uniform cylinder of radius R is spinned about its axis to the angular velocity ω_0 and then placed into the contact of a rough vertical wall. The coefficient of friction between the vertical wall and the cylinder as well as between the horizontal surface is equal to 1/3. How many turns the cylinder rotate before it stops? (Take $g = 10$ m s^{-2})

(a) $2\pi\dfrac{\omega_0^2}{R}$ (b) $\dfrac{4}{3}\dfrac{\omega_0^2}{R}$

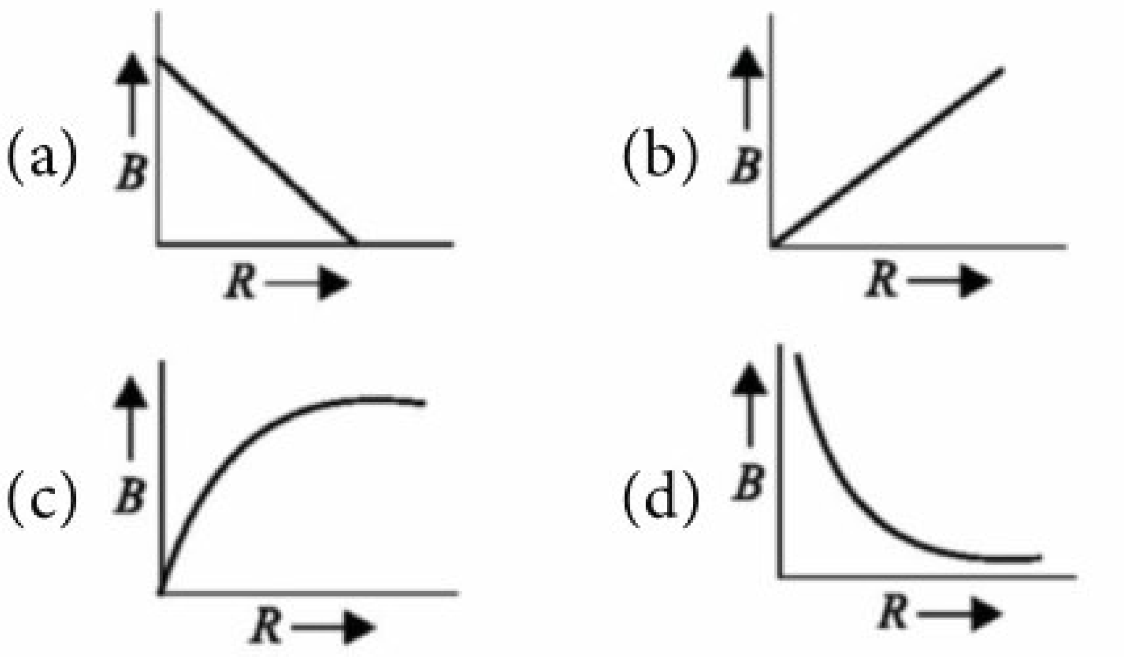
(c) $\dfrac{\omega_0^2 R}{32\pi}$ (d) $\dfrac{1}{40\pi}\omega_0^2 R$

2. A ball is projected from a point A on a smooth inclined plane which makes an angle α to the horizontal. The velocity of projection makes an angle θ with the plane upwards. If on the second bounce the ball is moving perpendicular to the plane, find e in terms of α and θ. Here e is the coefficient of restitution between the ball and the plane.

(a) $2\sin\alpha\cos\theta$ (b) $\dfrac{1}{2}\cot\alpha\cot\theta - 1$

(c) $\dfrac{3}{4}\sin\alpha\cot\theta + 1$ (d) $\dfrac{1}{2}\cot\alpha\tan\theta$

3. A charge Q is uniformly distributed over the surface of non-conducting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω. As a result of this rotation a magnetic field of induction B is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc then the variation of the magnetic induction at the centre of the disc will be represented by the figure



4. Two batteries of emf ε_1 and ε_2 having internal resistance r_1 and r_2 respectively are connected in series to an external resistance R. Both the batteries are getting discharged. The above described combination of these two batteries has to produce a weaker current than when any one of the batteries is

By Akhil Tewari, Author of Rank up Physics (JEE Main & Advanced), Director of School Toppers, Thane, Maharashtra, Ph.: +917900025559

connected to the same resistor. For this requirement to be fulfilled

(a) $\frac{\varepsilon_2}{\varepsilon_1}$ must not lie between $\frac{r_2}{r_1+R}$ and $\frac{r_1}{r_2+R}$

(b) $\frac{\varepsilon_2}{\varepsilon_1}$ must not lie between $\frac{r_2}{r_1+R}$ and $\frac{r_2+R}{r_1}$

(c) $\frac{\varepsilon_2}{\varepsilon_1}$ must lie between $\frac{r_2}{r_1+R}$ and $\frac{r_1}{r_2+R}$

(d) $\frac{\varepsilon_2}{\varepsilon_1}$ must lie between $\frac{r_2}{r_1+R}$ and $\frac{r_2+R}{r_1}$.

NUMERICAL VALUE TYPE

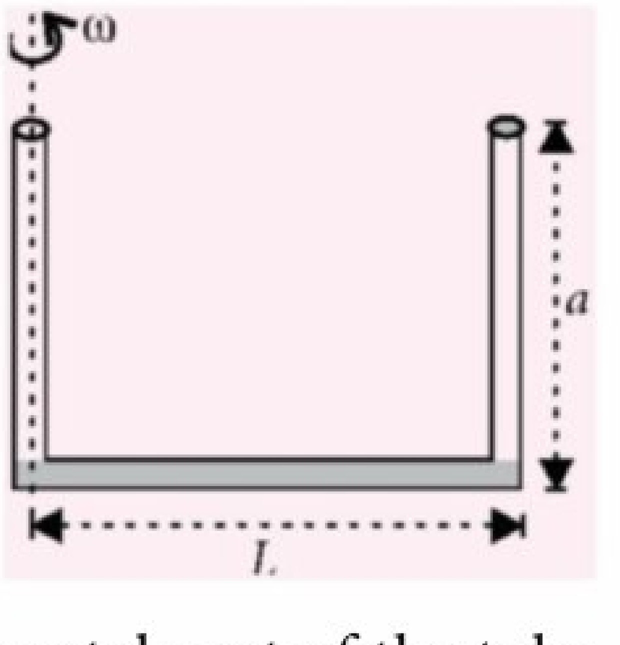
5. A rod of uniform cross-section of mass M and length l is hinged about an end to swing freely in a vertical plane. However, its density is non-uniform and varies linearly from hinged end to the free end doubling its value. The moment of inertia of the rod, about the rotation axis passing through the hinge point is xMl^2. Find the value of x.

6. Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m s^{-1}, the focal length of the lens (in cm) is
(a) 10 cm (b) 15 cm
(c) 20 cm (d) 30 cm

7. A uniform rod of length $l = 12$ cm lies on a smooth horizontal table. A particle moving on the table strikes the rod perpendicularly at an end and stops. Find the distance, travelled (in m) by the centre of the rod by the time it turns through an angle $\frac{\pi}{2}$.

SUBJECTIVE TYPE

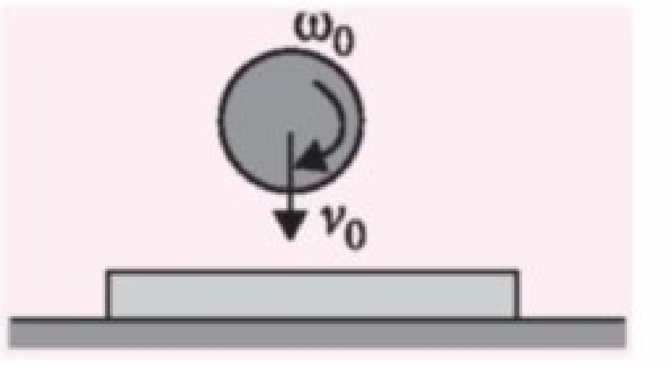
8. Length of a horizontal arm of a U-tube is L and ends of both the vertical arms are open to atmospheric pressure P_o. A liquid of density ρ is poured in the tube such that liquid just fills the horizontal part of the tube as shown in figure.



Now one end of the opened ends is sealed and the tube is then rotated about a vertical axis passing through the other vertical arm with angular speed ω. If length of each vertical arm is a and in the sealed end liquid rises to a height y, find pressure in the sealed tube during rotation.

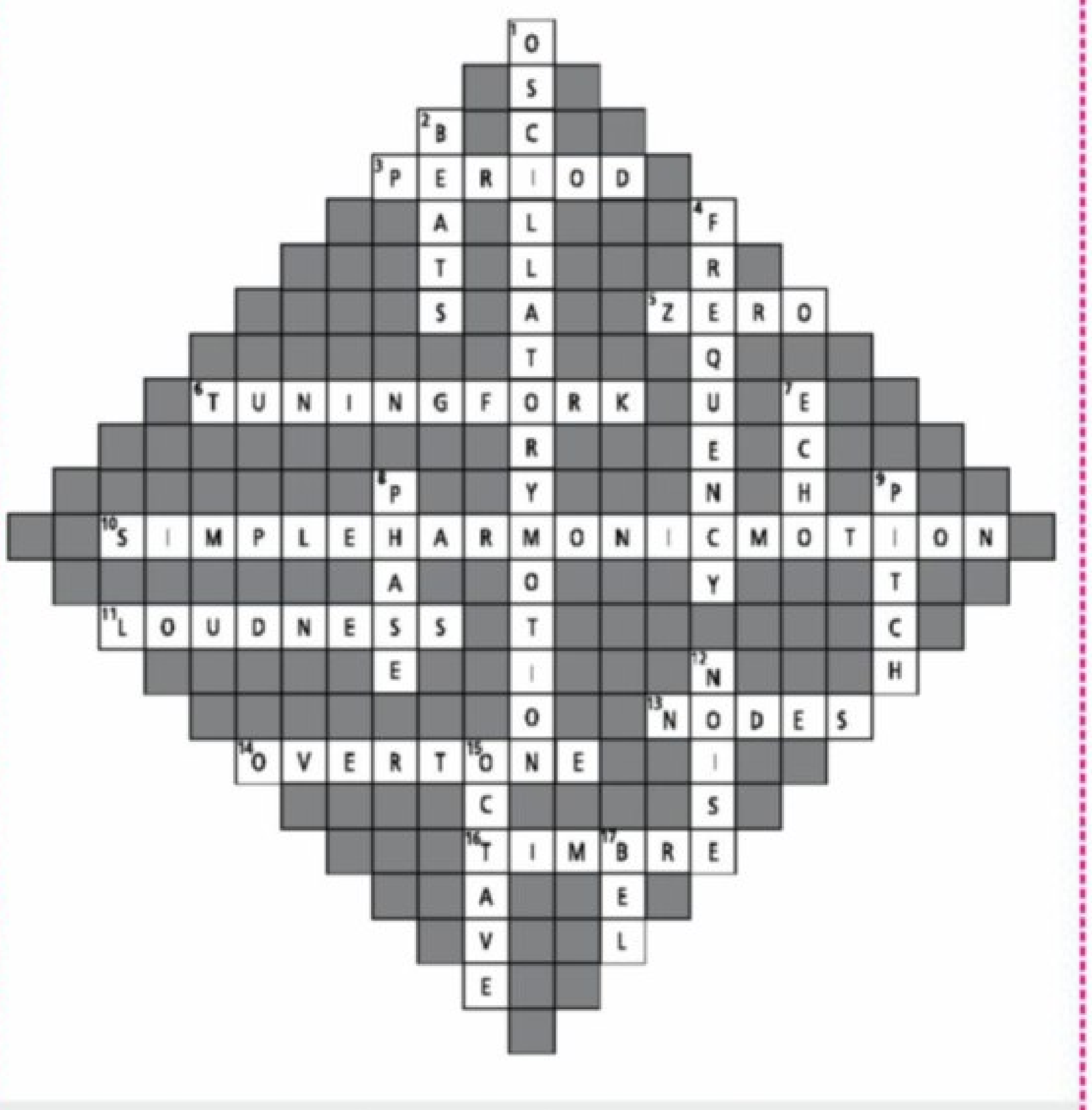
9. A conical cup of height b, semi vertical angle α rests open end down on a flat surface. The cup is filled to height H ($H < b$) with liquid of density ρ. What is the upward lifting force on the cup?

10. A spherical ball of radius r and mass m collides with a plank of mass M kept on a smooth horizontal surface.



Before impact, the centre of the ball has a velocity v_0 and angular velocity ω_0 as shown. The normal velocity is reversed with same magnitude and the ball stops rotating after the impact. Find the distance on the plank between first two impacts of the ball. The coefficient of friction between the ball and the plank is μ. Assume plank is large enough.

CROSSWORD SOLUTION SEPTEMBER 2019



Solution Senders

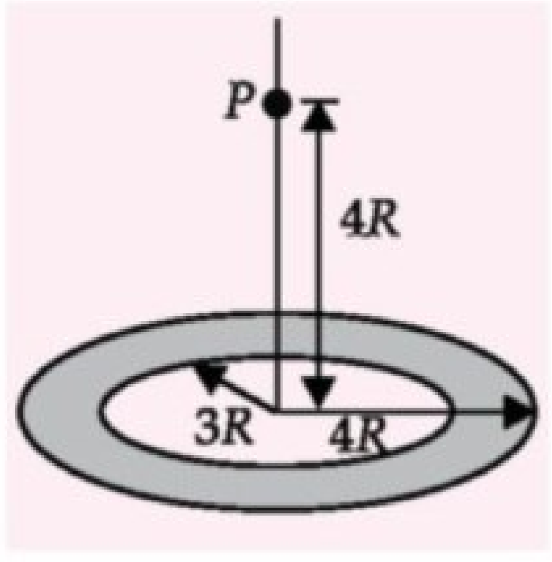
1. Sameekshya Kar, Odisha
2. Hariprashath, Pondicherry
3. Nitin Kumar, Mumbai
4. Devjit Acharjee, Kolkata
5. Rajashree Tekaday

GEAR UP FOR JEE MAIN 2020

Between 6th to 11th January, 2020

with Numerical Value Type Questions

1. A thin uniform annular disc (see figure) of mass M has outer radius $4R$ and inner radius $3R$. The work required to take a unit mass from point P on its axis to infinity is



(a) $\frac{2GM}{7R}(4\sqrt{2}-5)$ (b) $-\frac{2GM}{7R}(4\sqrt{2}-5)$

(c) $\frac{GM}{4R}$ (d) $\frac{2GM}{5R}(\sqrt{2}-1)$

2. Four moles of carbon monoxide are mixed with four moles of carbon dioxide. Assuming the gases to be ideal, the ratio of specific heats of the mixture is

(a) $\frac{15}{11}$ (b) $\frac{46}{30}$ (c) $\frac{5}{3}$ (d) $\frac{7}{4}$

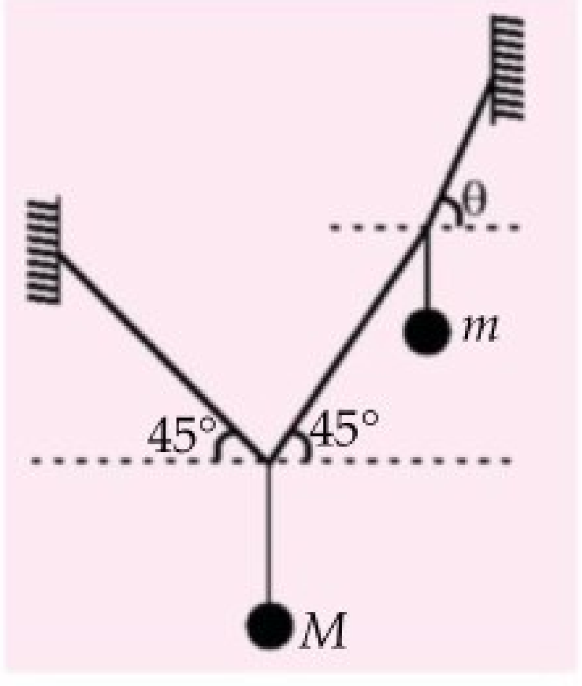
3. The density of a solid at normal pressure is ρ. When the solid is subjected to an excess pressure P, the density changes to ρ'. If the bulk modulus of the solid is K, then the ratio $\frac{\rho'}{\rho}$ is

(a) $1+\frac{P}{K}$ (b) $1+\frac{K}{P}$ (c) $\frac{P}{P+K}$ (d) $\frac{K}{P+K}$

4. A radioactive substance is being produced at a constant rate of 200 nuclei per second. The decay constant of the substance is 1 s^{-1}. After what time, the number of radioactive nuclei will become 100? Initially, there are no nuclei present.

(a) 1 s (b) 2 s (c) ln(2) s (d) $\frac{1}{\ln(2)}$ s

5. Two masses m and M are attached to strings as shown in the figure. In equilibrium, $\tan\theta$ is



(a) $1+\frac{2M}{m}$

(b) $1+\frac{2m}{M}$

(c) $1+\frac{M}{2m}$ (d) $1+\frac{m}{2M}$

6. A rocket is launched vertically from the surface of the earth with an initial velocity v. How far above the surface of earth it will go? Neglect the air resistance. (Here R is the radius of the earth)

(a) $R\left(\frac{2gR}{v^2}-1\right)^{-1/2}$ (b) $R\left(\frac{2gR}{v^2}-1\right)$

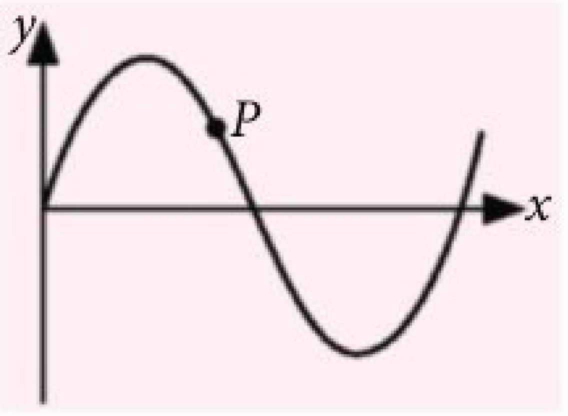
(c) $R\left(\frac{2gR}{v^2}-1\right)^{-1}$ (d) $R\left(\frac{2gR}{v^2}-1\right)^{2}$

7. A uniform magnetic field B = 1.2 mT is directed vertically upward throughout the volume of a laboratory chamber. A proton ($m_p = 1.67 \times 10^{-27}$ kg) enters the laboratory horizontally from South to North. Calculate the magnitude of centripetal acceleration of the proton if its speed is 3×10^7 m s^{-1}.

(a) 3.45×10^{12} m s^{-2} (b) 1.67×10^{12} m s^{-2}

(c) 5.25×10^{12} m s^{-2} (d) 2.75×10^{12} m s^{-2}

8. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm s^{-1}. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap-shot of the wave is shown in figure. The velocity of particle P when its displacement is 5 cm is



(a) $\frac{\sqrt{3}\pi}{50}\hat{j}$ m s^{-1} (b) $-\frac{\sqrt{3}\pi}{50}\hat{j}$ m s^{-1}

(c) $\frac{\sqrt{3}\pi}{50}\hat{i}$ m s^{-1} (d) $-\frac{\sqrt{3}\pi}{50}\hat{i}$ m s^{-1}

9. When a certain metallic surface is illuminated with monochromatic light of wavelength λ, the stopping potential for photoelectric current is $3V_0$ and the stopping potential changes to V_0 if it is illuminated by light of wavelength 2λ. Find the threshold wavelength.

(a) 6λ (b) $\frac{4\lambda}{3}$ (c) 4λ (d) 8λ

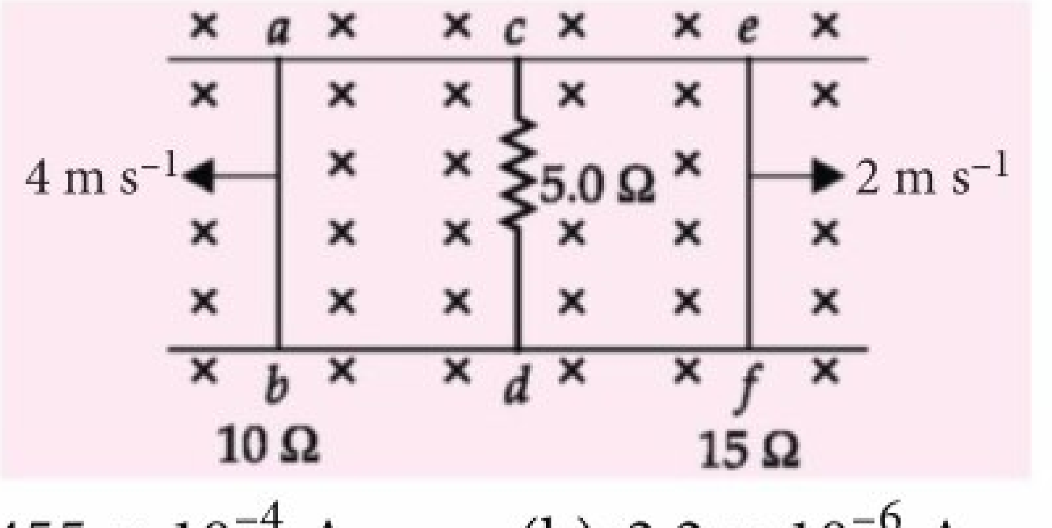
10. A boy is riding on a flat car of a train moving with velocity 10 m s^{-1}. The boy throws a ball which according to him makes angle 60° with the horizontal and in line with track. An observer on the ground observes the ball to rise vertically. How much does he see the ball rise?

(a) 10.5 m (b) 12 m (c) 15.3 m (d) 7.5 m

11. In a Young's double slit arrangement, a source of wavelength 6000 Å is used. The screen is placed 1 m from the slits. Fringes, formed on the screen, are observed by a student sitting close to the slits. The students eye, can distinguish two neighbouring fringes. Calculate the position of 3^{rd} bright and 5^{th} dark fringe from the centre of the screen.

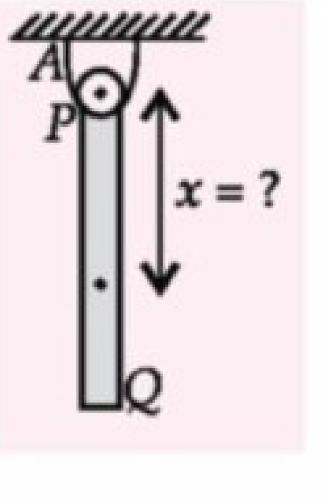
(a) 1.5 mm, 2.2 mm (b) 0.5 mm, 2.3 mm
(c) 0.874 mm, 1.31 mm (d) 2.75 mm, 2.22 mm

12. Two parallel rails with negligible resistance are 10 cm apart. They are connected by a 5 Ω resistor. The circuit also contains two metal rods having resistances of 10 Ω and 15 Ω along the rails. The rods are pulled away from the resistor at constant speed 4 m s^{-1} and 2 m s^{-1} respectively. A uniform magnetic field of magnitude 0.01 T is applied perpendicular to the plane of the rails. Find the current in the 5 Ω resistor.



(a) 1.455×10^{-4} A (b) 2.2×10^{-6} A
(c) 0.78×10^{-4} A (d) 2×10^{-5} A

13. A uniform rod *PQ* of mass 0.1 kg and length 0.12 m can swing in a vertical plane about *A* as a pendulum. A particle of mass 0.2 kg is attached to the rod at a distance *x* from *A*. Find *x* such that the period of vibration is minimum.



(a) 6.37 cm (b) 3.40 cm (c) 2.75 cm (d) 4.38 cm

14. For an amplitude modulated wave, the maximum amplitude is found to be 10 V while the minimum amplitude is found to be 2 V. What is the modulation index?

(a) $\frac{6}{7}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

15. A room is maintained at 20°C by a heater of resistance 20 ohm connected to 200 volt mains. The temperature is uniform throughout the room and the heat is transmitted through a glass window of area 1 m^2 and thickness 0.2 cm. Calculate the temperature outside. Thermal conductivity of glass is 0.2 cal $m^{-1}s^{-1}$ °C^{-1} and mechanical equivalent of heat is 4.2 J cal^{-1}.

(a) 20.02 °C (b) 15.24 °C
(c) 13.25 °C (d) 7.52 °C

16. A U-tube containing a liquid moves with a horizontal acceleration *a* along a direction joining the two vertical limbs. The separation between these limbs is *d*. The difference in their liquid levels is

(a) $\frac{ad}{g}$ (b) $\frac{2da}{g}$
(c) $\frac{da}{2g}$ (d) $d\tan\left(\frac{a}{g}\right)$

17. A circular coil of radius 1 cm, 500 turns and resistance 2 Ω is placed with its plane perpendicular to the horizontal component of the earth's magnetic field. It is rotated about its vertical diameter through 180° in 0.25 s. The current induced in the coil is (Horizontal component of the earth's magnetic field at the place is 3.0×10^{-5} T.)

(a) 1.9×10^{-3} A (b) 2.9×10^{-3} A
(c) 3.9×10^{-3} A (d) 4.9×10^{-3} A

18. A pure resistive circuit element *X* when connected to an ac supply of peak voltage 200 V gives a peak current of 5 A which is in phase with the voltage. A second circuit element *Y*, when connected to the same ac supply also gives the same value of peak current but the current lags behind by 90°. If the series combination of *X* and *Y* is connected to the same supply, what will be the rms value of current?

(a) $\frac{10}{\sqrt{2}}$ A (b) $\frac{5}{\sqrt{2}}$ A (c) $\frac{5}{2}$ A (d) 5 A

19. A tuning fork of frequency 512 Hz makes 4 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was

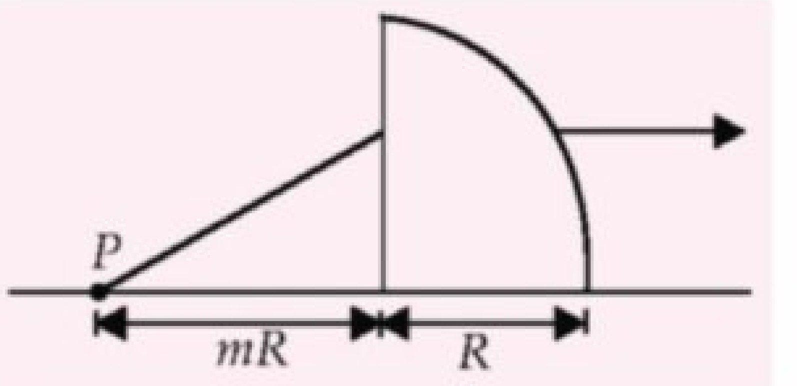
(a) 510 Hz (b) 514 Hz (c) 516 Hz (d) 508 Hz

20. The dimension of $\frac{1}{2}\varepsilon_0 E^2$, where ε_0 is permittivity of free space and E is electric field, is

(a) $[ML^2T^{-2}]$ (b) $[ML^{-1}T^{-2}]$
(c) $[ML^2T^{-1}]$ (d) $[MLT^{-1}]$

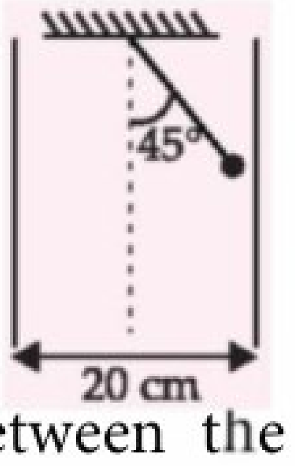
Numerical Value Type

21. A quarter cylinder of radius R and refractive index 1.5 is placed on a table. A point object P is kept at a distance mR from it. Find the value of m for which a ray from P will emerge parallel to the table as shown in figure.



22. A flat thin circular disc has a radius 4 cm and a circular hole of radius $\frac{1}{2}$ cm is made in it with its centre at a distance of 1 cm from the centre of the disc. The mass of the disc is 10 kg. If the moment of inertia of the system about an axis passing through the centre of the hole is $N \times 10^{-3}$ kg m^2, find the value of N.

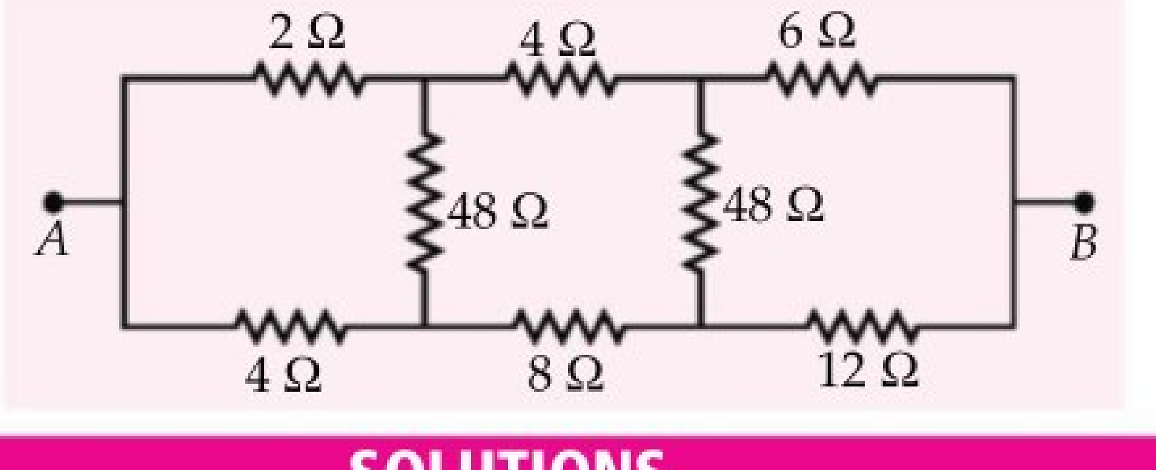
23. A small sphere of mass 2.0 g and having charge 0.5 mC is suspended by a string between the plates of a parallel plate capacitor as shown in the figure.



What potential difference (in volt) between the plates (separation 20 cm) should be applied so that the string makes an angle of 45° with the vertical? (Take g = 10 m s^{-2})

24. A ball is allowed to fall freely from a height of 3 metres on to a fixed plate. The successive rebound heights are $h_1, h_2, h_3, \ldots$. If the distance covered by the ball before coming to rest is x metres, find the value of x. (Given that the coefficient of restitution is 0.5)

25. Find the resistance (in Ω) between the terminals A and B of the network shown below.

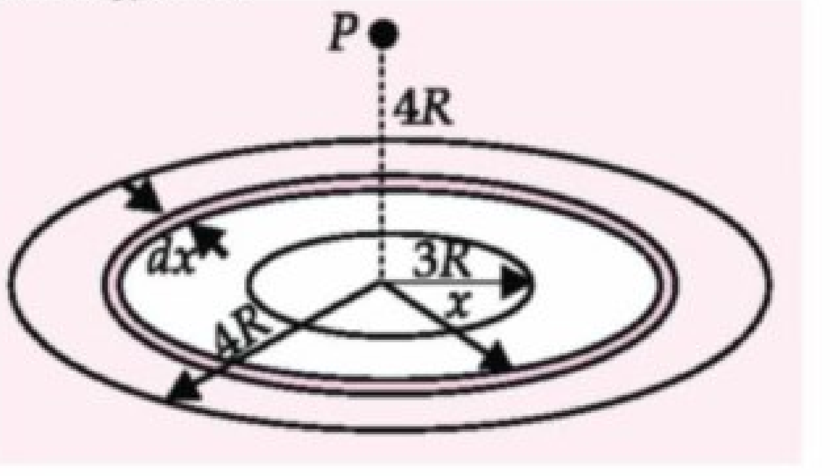


SOLUTIONS

1. **(a):** Mass per unit area of the disc,

$$\sigma = \frac{\text{Mass}}{\text{Area}} = \frac{M}{\pi\left((4R)^2 - (3R)^2\right)} = \frac{M}{7\pi R^2}$$

Consider a ring of radius x and thickness dx as shown in the figure.



Mass of the ring, $dM = \sigma 2\pi x dx = \frac{2\pi M x dx}{7\pi R^2}$

Potential at point P due to annular disc is

$$V_P = \int_{3R}^{4R} -\frac{G dM}{\sqrt{(4R)^2 + (x)^2}} = -\frac{GM 2\pi}{7\pi R^2}\int_{3R}^{4R} \frac{x dx}{\sqrt{16R^2 + x^2}}$$

Solving, we get

$$V_P = -\frac{GM 2\pi}{7\pi R^2}\left[\sqrt{16R^2 + x^2}\right]_{3R}^{4R} = -\frac{2GM}{7R}\left(4\sqrt{2} - 5\right)$$

Work done in moving a unit mass from P to ∞ =

$$V_\infty - V_P = 0 - \left(\frac{-2GM}{7R}\left(4\sqrt{2} - 5\right)\right) = \frac{2GM}{7R}\left(4\sqrt{2} - 5\right)$$

2. **(a):** The ratio of specific heats of a mixture is

$$\gamma_{\text{mixture}} = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 C_{V_1} + n_2 C_{V_2}}$$

C_V and C_P for diatomic gas are $5R/2$ and $7R/2$ respectively, whereas those for a polyatomic gas are $3R$ and $4R$.

Here, $n_1 = 4$, $n_2 = 4$

$$C_{P_1} = \frac{7}{2}R, C_{V_1} = \frac{5}{2}R \quad \text{or} \quad C_{P_2} = 4R, C_{V_2} = 3R$$

$$\therefore \quad \gamma_{\text{mixture}} = \frac{4 \times \frac{7}{2}R + 4 \times 4R}{4 \times \frac{5}{2}R + 4 \times 3R} = \frac{30}{22} = \frac{15}{11}$$

3. **(a):** Density, $\rho = \frac{m}{V}$

$$\therefore \quad \frac{d\rho}{\rho} = -\frac{dV}{V} \quad \text{...(i)}$$

Bulk modulus, $K = -\frac{P}{dV/V}$

Substituting (i) in the expression of the bulk modulus we get

$$K = \frac{P}{\frac{d\rho}{\rho}} \quad \text{or} \quad \frac{d\rho}{\rho} = \frac{P}{K} \quad \text{...(ii)}$$

Since increase of pressure increases the density,

$\therefore \quad d\rho = \rho' - \rho$ ...(iii)

From (ii) and (iii), we get

$$\frac{\rho' - \rho}{\rho} = \frac{P}{K} \quad \text{or} \quad \frac{\rho'}{\rho} = 1 + \frac{P}{K}$$

4. **(c) :** Let N be the number of nuclei at any time t. Then,

$$\frac{dN}{dt} = 200 - \lambda N$$

or $\int_0^N \frac{dN}{(200-\lambda N)} = \int_0^t dt \Rightarrow -\frac{1}{\lambda}[\ln(200-\lambda N)]_0^N = t$

$\ln(200 - \lambda N) - \ln(200) = -\lambda t$

$\ln\left(\frac{200-\lambda N}{200}\right) = -\lambda t$ or $\frac{200-\lambda N}{200} = e^{-\lambda t}$

$1 - \frac{\lambda N}{200} = e^{-\lambda t}$ or $N = \frac{200}{\lambda}(1 - e^{-\lambda t})$

As $N = 100$ and $\lambda = 1\ s^{-1}$,

$\therefore\ 100 = 200(1 - e^{-t})$ or $e^{-t} = \frac{1}{2}\ \therefore t = \ln(2)\ s$

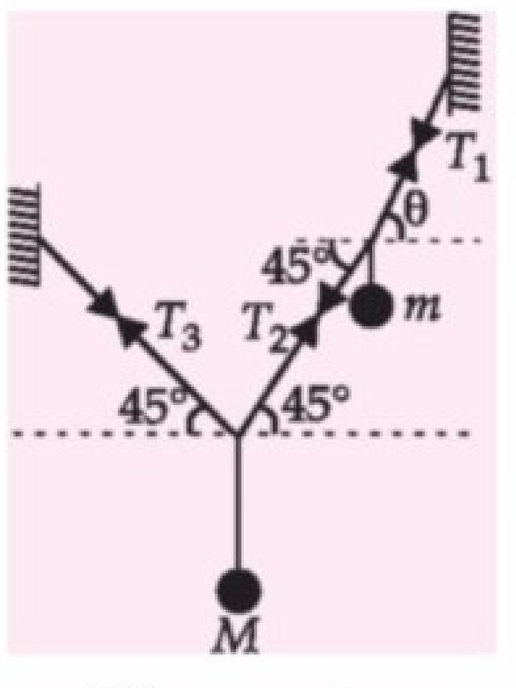
5. **(b):** In equilibrium,

$T_1\cos\theta = T_2\cos 45°$ ...(i)

$T_2\cos 45° = T_3\cos 45°$ ...(ii)

$T_1\sin\theta = T_2\sin 45° + mg$ ...(iii)

$T_2\sin 45° + T_3\sin 45° = Mg$ ...(iv)



From (ii) and (iv), we get

$$T_2 = T_3 = \frac{Mg}{\sqrt{2}}$$

Substituting this value of T_2 in eqn. (i), we get

$$T_1\cos\theta = \frac{Mg}{2}, \quad \text{...(v)}$$

Substituting the value of T_2 in eqn. (iii), we get

$$T_1\sin\theta = \frac{Mg}{2} + mg \quad \text{...(vi)}$$

Divide eqn. (vi) by eqn. (v), we get

$$\tan\theta = \left(1 + \frac{2m}{M}\right)$$

6. **(c) :** On the surface of earth,

Total energy = Kinetic energy + Potential energy

$$= \frac{1}{2}mv^2 - \frac{GmM}{R}$$

At the highest point, $v = 0$,

Potential energy = $-\frac{GmM}{(R+h)}$

where h is the maximum height.

According to the law of conservation of mechanical energy, we get

$\frac{1}{2}mv^2 - \frac{GmM}{R} = -\frac{GmM}{R+h}$ or $\frac{1}{2}v^2 = \frac{GMh}{(R)(R+h)}$

$\frac{1}{2}v^2 = \frac{gRh}{R+h}$ $\quad \left(\because g = \frac{GM}{R^2}\right)$

or $\frac{R+h}{h} = \frac{2gR}{v^2}$ or $h = R\left(\frac{2gR}{v^2} - 1\right)^{-1}$

7. **(a) :** Here,

Magnetic field, $B = 1.2\ mT = 1.2 \times 10^{-3}\ T$

Mass of the proton, $m_p = 1.67 \times 10^{-27}$ kg

Speed of the proton, $v = 3 \times 10^7\ m\ s^{-1}$

When an proton enters a uniform magnetic field perpendicular to its velocity, it follows a circular path and whose radius is given by

$$\therefore\ R = \frac{m_p v}{qB} \quad \text{...(i)}$$

Centripetal acceleration of the proton is

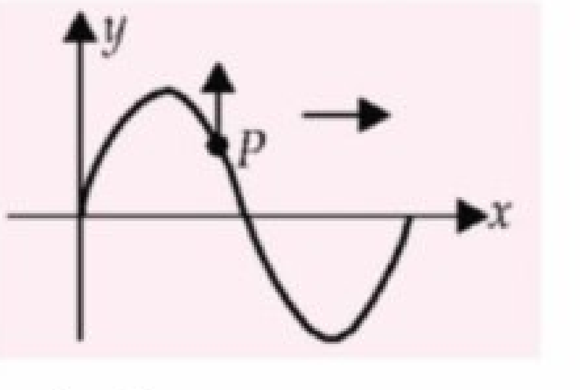
$$a_c = \frac{v^2}{R} = \frac{v^2}{\left(\frac{m_p v}{qB}\right)} \quad \text{(Using (i))}$$

$$= \frac{vqB}{m_p}$$

$$= \frac{(3\times10^7\ m\ s^{-1})\times(1.6\times10^{-19}\ C)\times(1.2\times10^{-3}\ T)}{(1.67\times10^{-27}\ kg)}$$

$= 3.45 \times 10^{12}\ m\ s^{-2}$

8. **(a) :** Particle velocity v_P is related to the displacement of the particle from the mean position as



$$v_P = 2\pi\upsilon\sqrt{A^2 - y^2} \text{ or } v_P = 2\pi\left(\frac{v}{\lambda}\right)\sqrt{A^2 - y^2}$$

$$= \frac{2\pi}{0.5}\times 0.1\sqrt{(0.1)^2 - (0.05)^2} = \frac{\sqrt{3}\pi}{50}\hat{j}\ m\ s^{-1}$$

Since the wave is travelling in $+x$ direction, the particle P shown on the snap-shot must have its velocity in $+y$ direction at the moment of time given.

9. **(c) :** According to Einstein's photoelectric equation

$$3V_0 e = \frac{hc}{\lambda} - W_0 \quad \text{...(i)}$$

$$V_0 e = \frac{hc}{2\lambda} - W_0 \quad \text{...(ii)}$$

Multiplying (ii) by 3, we get

$$3V_0 e = \frac{3hc}{2\lambda} - 3W_0 \quad \text{...(iii)}$$

Comparing (i) and (iii), we get

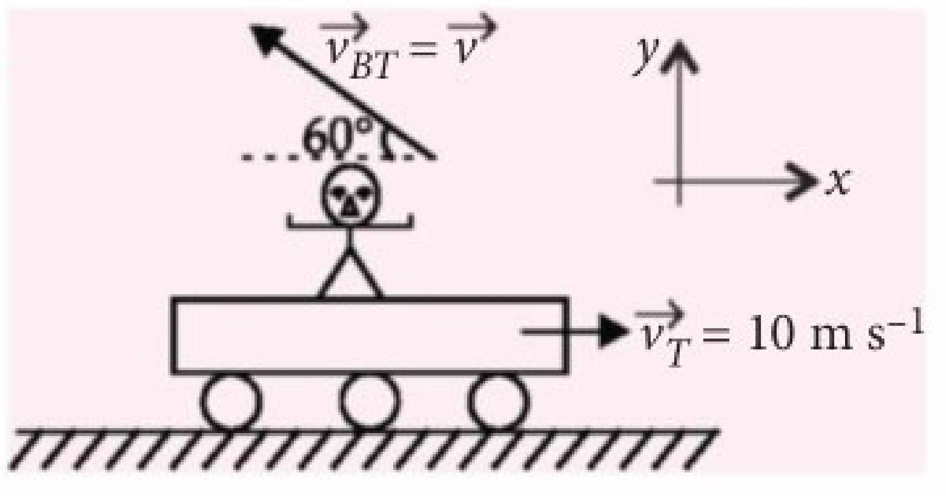
$$\frac{hc}{\lambda} - W_0 = \frac{3hc}{2\lambda} - 3W_0$$

$$W_0 = \frac{hc}{4\lambda}$$

If λ_0 is threshold wavelength, then $W_0 = \frac{hc}{\lambda_0}$

$\therefore\ \lambda_0 = 4\lambda$

10. (c) :



$$\vec{v}_{\text{ball, train}} = \vec{v}_{\text{ball}} - \vec{v}_{\text{train}}$$

$$\therefore\ \vec{v}_{\text{ball}} = \vec{v}_{\text{ball, train}} + \vec{v}_{\text{train}}$$

Since ball moves in vertical direction, its x-component must vanish.

Let the velocity of ball relative to train $\vec{v}_{BT} = \vec{v}$.

$\therefore\ v_x = v\cos 60° - v_T = 0$ ...(i)

and $v_y = v\sin 60°$ ...(ii)

From (i), $v = \frac{v_T}{\cos 60°} = 20\text{ m s}^{-1}$

and $v_y = 20\times\sin 60° = 10\sqrt{3} = 10\sqrt{3}\text{ m s}^{-1}$

$\therefore$ Maximum height reached by ball $= \frac{v_y^2}{2g}$

$$= \frac{(10\sqrt{3})^2}{2\times 9.8} \approx 15.3\text{ m}$$

11. (c) : For angle more than 1′, angular fringe width

$$\theta_0 = \frac{\beta}{D} = \frac{\lambda}{d} \geq \frac{\pi}{180\times 60}\quad\left(\text{As } 1' = \frac{\pi}{180\times 60}\text{ rad}\right)$$

i.e., $d \leq \frac{6\times 10^{-7}\times 180\times 60}{\pi}$ or $d_{\max} = 2.06\text{ mm}$

Now as the position of a fringe on the screen relative to central maxima is

$$y = \frac{D}{d}(\Delta x)$$

So for 3rd bright fringe as $\Delta x = n\lambda = 3\lambda$,

$$\therefore\ y_{3B} = \frac{1}{(2.06\times 10^{-3})}\times 3\times 6\times 10^{-7} = 0.874\text{ mm}$$

And for 5th dark fringe, as

$$\Delta x = (2n-1)\frac{\lambda}{2} = \frac{9}{2}\lambda$$

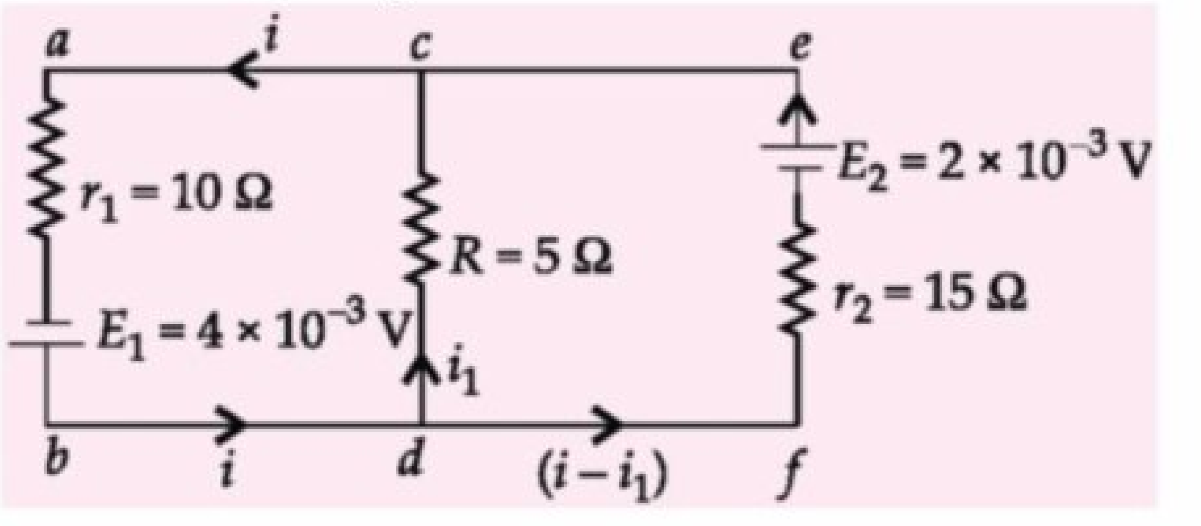
$$\therefore\ y_{5D} = \frac{1}{2.06\times 10^{-3}}\times\frac{9}{2}\times 6\times 10^{-7} = 1.31\text{ mm}$$

12. (a): Two moving conductors will act as a source of emf. For rod ab, $E_1 = Bvl = (0.01)\ (4)\ (0.1) = 4\times 10^{-3}$ V and internal resistance $r_1 = 10\ \Omega$.

Similarly, rod ef will also act as a source of emf, $E_2 = (0.01)\ (2)\ (0.1) = 2\times 10^{-3}$ V and internal resistance, $r_2 = 15\ \Omega$.

From Fleming's right hand rule we can determine the polarity of source of induced emf,

$V_b > V_a$ and $V_e > V_f$.



From KVL in loop $abdca$, we have

$4\times 10^{-3} - 5i_1 - 10i = 0$ ...(i)

and in loop $cdfec$, we have

$-15(i - i_1) + 2\times 10^{-3} + 5i_1 = 0$ ...(ii)

On solving equations (i) and (ii), we get

$i_1 = 1.455\times 10^{-4}$ A.

13. (c)

14. (b): Let A_c and A_m be the amplitude of carrier wave and message signal wave.

Given,

$A_{\max} = A_c + A_m = 10$ V

$A_{\min} = A_c - A_m = 2$ V

On solving, we get, $A_c = 6$ V, $A_m = 4$ V

Modulation index,

$$\mu = \frac{A_m}{A_c} = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} = \frac{10-2}{10+2} = \frac{8}{12} = \frac{2}{3}$$

15. (b): Heat produced by the heater per second is

$$\frac{Q}{t} = \frac{V^2}{R} = \frac{(200)^2}{20} = 2000\text{ J s}^{-1}$$

$$= \frac{2000}{4.2}\text{ cal s}^{-1} \quad \text{...(i)}$$

Heat transmitted through the window per second is

$$\frac{Q}{t} = \frac{KA(T_1 - T_2)}{d}$$

where, T_2 = temperature outside.

Given $T_1 = 20°$C, $d = 0.2$ cm, $A = 1\text{ m}^2 = 10^4\text{ cm}^2$ and $K = 0.2\text{ cal m}^{-1}\text{s}^{-1}\,°\text{C}^{-1} = 0.002\text{ cal cm}^{-1}\text{s}^{-1}\,°\text{C}^{-1}$.

Thus

$$\frac{Q}{t} = \frac{0.002\times 10^4\times(20 - T_2)}{0.2} = 100(20 - T_2)\text{ cal s}^{-1} \quad \text{...(ii)}$$

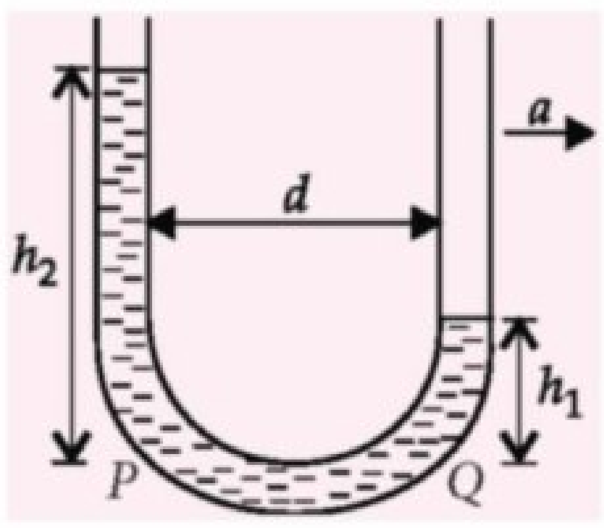
Equating (i) and (ii), we have

$$100(20 - T_2) = \frac{2000}{4.2}$$

$$(20 - T_2) = \frac{20}{4.2}$$

which gives $T_2 = 15.24°C$.

16. (a): Let A = area of cross-section of the tube, ρ = density of the liquid



Consider the section PQ of the tube.

Mass of the liquid in $PQ = dA\rho$

Pressure at $P = P_0 + h_2\rho g$

where P_0 is the atmospheric pressure

Pressure at $Q = P_0 + h_1\rho g$

Net force to the right on $PQ = (h_2\rho g - h_1\rho g)A$

$\therefore \ (h_2 - h_1)\rho gA = (dA\rho)a$ or $(h_2 - h_1)g = da$

or $\ h_2 - h_1 = \dfrac{ad}{g}$

17. (a): Initial magnetic flux through the coil,

$\phi_i = BA\cos\theta = 3.0 \times 10^{-5} \times (\pi \times 10^{-2}) \times \cos 0°$

$= 3\pi \times 10^{-7}$ Wb

Final magnetic flux after the rotation

$\phi_f = 3.0 \times 10^{-5} \times (\pi \times 10^{-2}) \times \cos 180°$

$= -3\pi \times 10^{-7}$ Wb

$$\text{Induced emf, } \varepsilon = -N\frac{d\phi}{dt} = -\frac{N(\phi_f - \phi_i)}{t}$$

$$= -\frac{500(-3\pi\times10^{-7} - 3\pi\times10^{-7})}{0.25}$$

$$= \frac{500\times(6\pi\times10^{-7})}{0.25} = 3.8 \times 10^{-3}\text{ V}$$

$$I = \frac{\varepsilon}{R} = \frac{3.8\times10^{-3}\text{ V}}{2\ \Omega} = 1.9\times10^{-3}\text{ A}$$

18. (c): As current is in phase with the applied voltage, X must be R.

$$R = \frac{V_0}{I_0} = \frac{200\text{V}}{5\text{A}} = 40\ \Omega$$

As current lags behind the voltage by 90°, Y must be an inductor.

$$X_L = \frac{V_0}{I_0} = \frac{200\text{V}}{5\text{A}} = 40\ \Omega$$

In the series combination of X and Y,

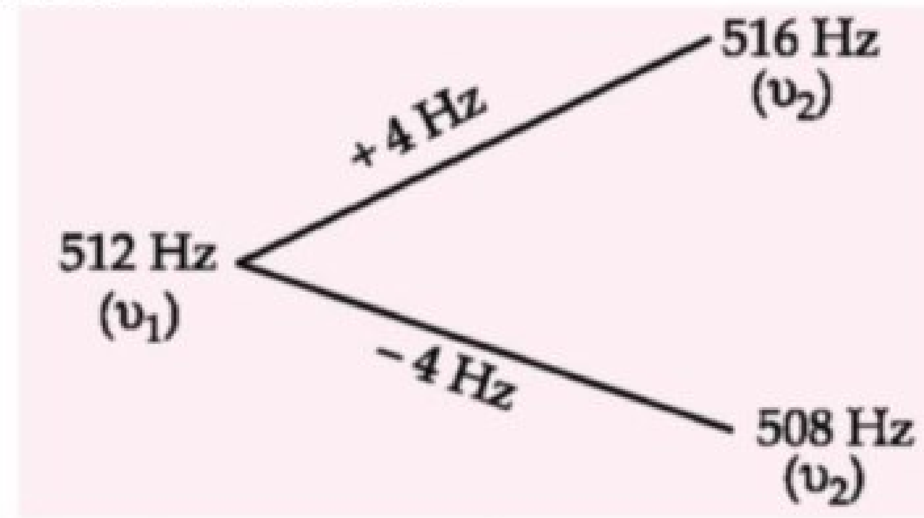
$$Z = \sqrt{R^2 + X_L^2} = \sqrt{40^2 + 40^2} = 40\sqrt{2}\ \Omega$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_0}{\sqrt{2}Z} = \frac{200}{\sqrt{2}(40\sqrt{2})} = \frac{5}{2}\text{A}$$

19. (d): Let the frequencies of tuning fork and piano string be υ_1 and υ_2 respectively.

$\therefore \ \upsilon_2 = \upsilon_1 \pm 4 = 512 \text{ Hz} \pm 4$

$= 516$ Hz or 508 Hz



Increase in the tension of a piano string increases its frequency.

If $\upsilon_2 = 516$ Hz, further increase in υ_2, resulted in an increase in the beat frequency. But this is not given in the question.

If $\upsilon_2 = 508$ Hz, further increase in υ_2 resulted in decrease in the beat frequency. This is given in the question, where the beat frequency decreases to 2 beats per second.

Therefore, the frequency of the piano string before increasing the tension was 508 Hz.

20. (b): Energy density of an electric field E is

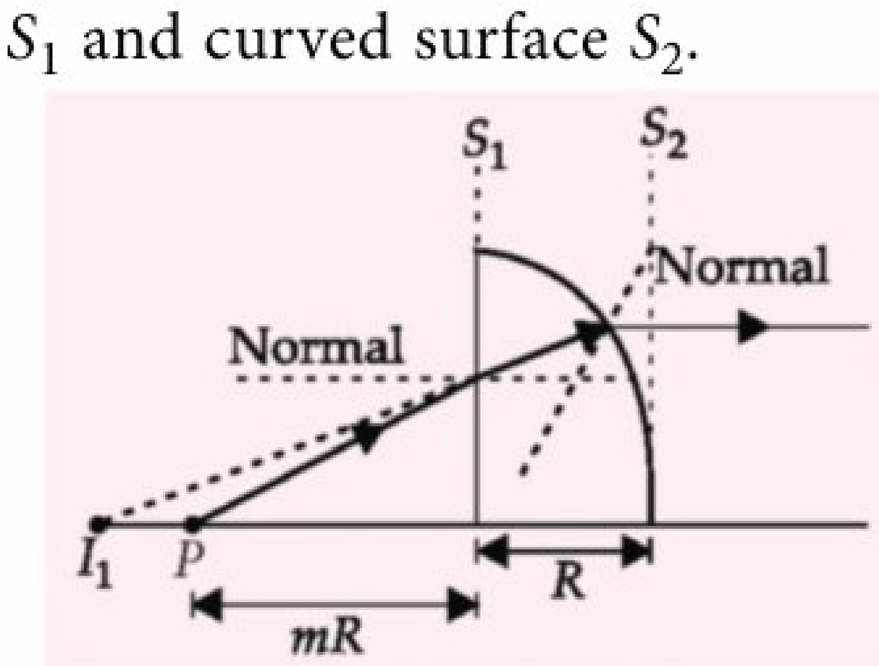
$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

where ε_0 is permittivity of free space.

$$[u_E] = \frac{\text{Energy}}{\text{Volume}} = \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{L}^3]} = [\text{ML}^{-1}\text{T}^{-2}]$$

Hence, the dimension of $\frac{1}{2}\varepsilon_0 E^2$ is $]\text{ML}^{-1}\text{T}^{-2}]$.

21. (1.33): We will consider two refractions at plane surface S_1 and curved surface S_2.



Refraction at S_1,

$$\frac{\mu}{v_1} - \frac{1}{(-mR)} = 0$$

$v_1 = -\mu mR$. The image is virtual and on left of surface S_1. It acts as an object for refraction at the curved surface.

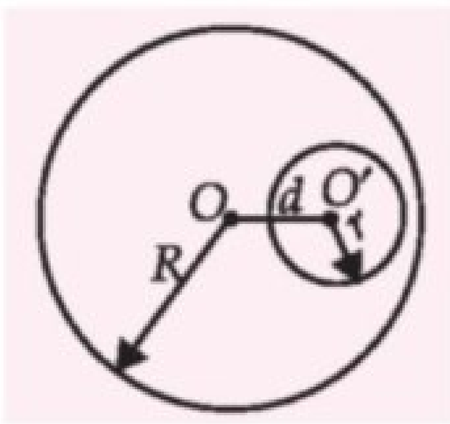
Refraction at S_2,

$$\frac{1}{v_2} - \frac{\mu}{-(m\mu R + R)} = \frac{1-\mu}{(-R)} \quad \text{or} \quad \frac{\mu}{(m\mu R + R)} = \frac{1-\mu}{(-R)}$$

as emergent rays are parallel to principal axis $v_2 = \infty$.
On solving for m, we get

$$m = \frac{1}{\mu^2 - \mu} = \frac{1}{\left(\frac{3}{2}\right)^2 - \frac{3}{2}} = \frac{4}{3}$$

22. (8.90) :



Let O be the centre of the disc of radius R and O' be centre of hole of radius r at a distance d from the centre of the disc.

The moment of inertia of the whole disc about an axis through O' is

$$I_1 = \frac{1}{2}MR^2 + Md^2$$

The moment of inertia of the removed portion of the disc about an axis through O' is

$$I_2 = \frac{1}{2}mr^2 = \frac{1}{2}M\frac{r^2}{R^2}r^2 \quad \left(\because m = \frac{M}{\pi R^2}\pi r^2\right)$$

$$= \frac{1}{2}\frac{Mr^4}{R^2}$$

The moment of inertia of the system about the given axis is

$$I = I_1 - I_2 = M\left[\frac{R^2}{2} + d^2 - \frac{r^4}{2R^2}\right]$$

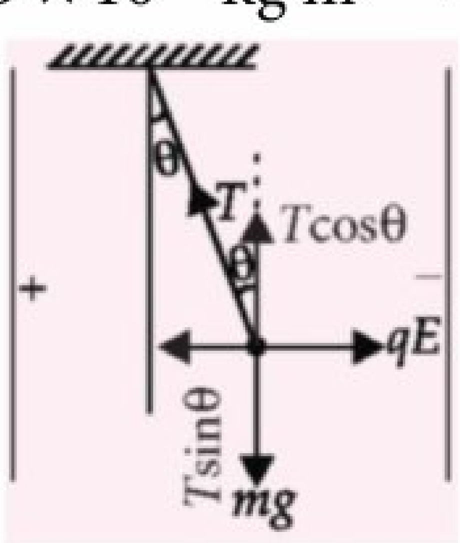
Here, $M = 10$ kg, $R = 4$ cm $= 4 \times 10^{-2}$ m,

$$r = \frac{1}{2}\text{ cm} = \frac{1}{2}\times 10^{-2}\text{ m}, d = 1\text{ cm} = 10^{-2}\text{ m}$$

$$\therefore \quad I = 10\left[\frac{16}{2}\times 10^{-4} + 1\times 10^{-4} - \frac{10^{-8}}{2\times 16\times 16\times 10^{-4}}\right]$$

$= 10(8 \times 10^{-4} + 10^{-4} - 0.002 \times 10^{-4})$
$= 8.9 \times 10^{-3}$ kg m^2 $\Rightarrow$ N = 8.9

23. (8.00) :



In equilibrium,

$T\sin\theta = qE$ ...(i)

$T\cos\theta = mg$ ...(ii)

Dividing eqn. (i) by eqn. (ii), we get

$$\tan\theta = \frac{qE}{mg}$$

Here, $\theta = 45°$

$$\therefore \quad \tan 45° = \frac{qE}{mg} \text{ or } 1 = \frac{qE}{mg}$$

Hence electric field $E = \frac{mg}{q}$

$\therefore$ Potential difference between the plate

$$V = Ed = \frac{mgd}{q}$$

Substituting the given values, we get

$$\text{or} \quad V = \frac{2\times 10^{-3}\times 10\times 20\times 10^{-2}}{0.5\times 10^{-3}} = \frac{4}{0.5} = 8.0\text{ V}$$

24. (5.00) : Let u be the velocity with which the ball strikes the plate and v be the velocity of rebound. Let H be the height from which the ball is dropped.

$$\therefore \quad u = \sqrt{2gH}$$

$$v = \sqrt{2gh_1}$$

Coefficient of restitution, $e = \frac{v}{u} = \sqrt{\frac{h_1}{H}}$

$$e^2 = \frac{h_1}{H} \text{ or } h_1 = e^2 H$$

Similarly, $h_2 = e^2 h_1 = e^4 H$ and so on.

Total distance travelled by the ball before it comes to rest

$x = H + 2h_1 + 2h_2 + 2h_3 + ...$
$= H + 2e^2H + 2e^4H + 2e^6H + ...$
$= H(1 + 2e^2 + 2e^4 + 2e^6 + ...)$
$= H[1 + 2e^2 (1 + e^2 + e^4 + ...)]$

$$= H\left[1 + \frac{2e^2}{1 - e^2}\right]$$

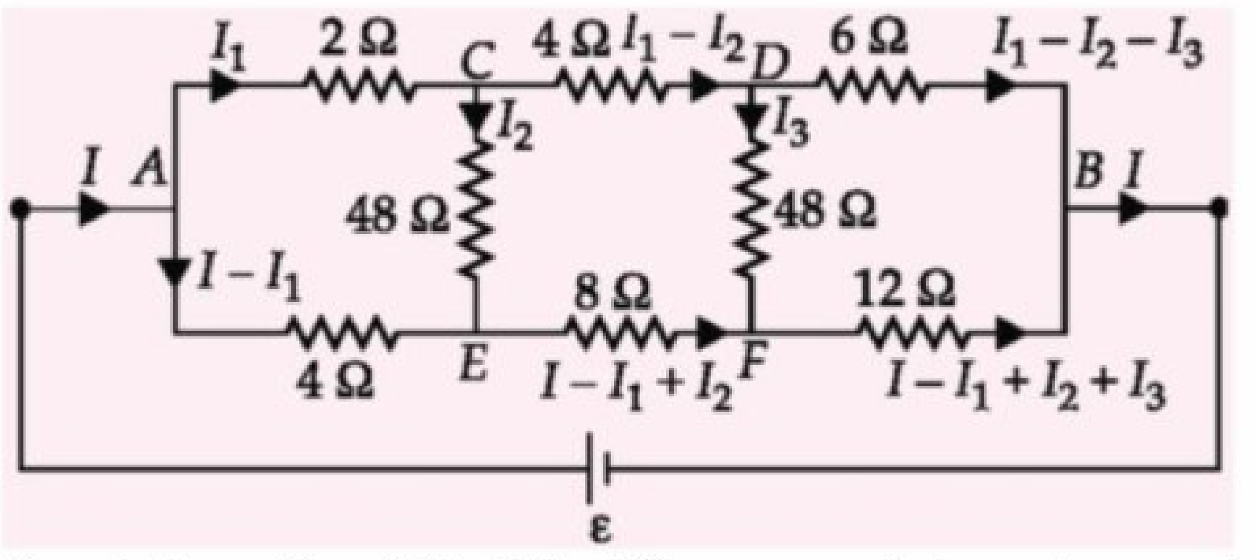
(Using that it is geometric series and $e < 1$)

$$x = H\left[\frac{1+e^2}{1-e^2}\right] = 3\left[\frac{1+0.5^2}{1-0.5^2}\right] = 5\text{ metres}$$

25. (8.00) : Let a battery of emf e connected between A and B. The distribution of currents in the various arms are as shown in figure.

Monthly Test Drive CLASS XII ANSWER KEY

1. (c)	**2.** (d)	**3.** (c)	**4.** (b)	**5.** (d)
6. (d)	**7.** (c)	**8.** (d)	**9.** (b)	**10.** (b)
11. (c)	**12.** (b)	**13.** (c)	**14.** (d)	**15.** (c)
16. (b)	**17.** (a)	**18.** (c)	**19.** (a)	**20.** (a,b,c)
21. (b,c)	**22.** (a,b)	**23.** (a,c,d)	**24.** (c,d)	**25.** (a,b,c)
26. (1)	**27.** (6)	**28.** (5)	**29.** (a)	**30.** (a)



Applying the Kirchhoff's second law in a closed loop $ACDBA$, we get

$$2I_1 + 4(I_1 - I_2) + 6(I_1 - I_2 - I_3) = \varepsilon$$

or $12I_1 - 10I_2 - 6I_3 = \varepsilon$

$$\therefore \quad 6I_1 - 5I_2 - 3I_3 = \frac{\varepsilon}{2} \quad \text{...(i)}$$

Applying the Kirchhoff's second law in a closed loop $AEFBA$, we get

$$4(I - I_1) + 8(I - I_1 + I_2) + 12(I - I_1 + I_2 + I_3) = \varepsilon$$

$$24I - 24I_1 + 20I_2 + 12I_3 = \varepsilon$$

$$6I - 6I_1 + 5I_2 + 3I_3 = \frac{\varepsilon}{4} \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$$6I = \frac{3}{4}\varepsilon \text{ or } \varepsilon = 8I \quad \text{...(iii)}$$

If R is the resistance of network between A and B then

$$\varepsilon = IR \quad \text{...(iv)}$$

From (iii) and (iv), we get

$$R = 8\ \Omega$$

◆◆

EXAM ALERT

JOINT ENTRANCE EXAMINATION MAIN – 2020

The National Testing Agency (NTA) will conduct the JEE Main-2020 Examination twice for admission to Undergraduate Programs in NITs, IIITs and other Centrally Funded Technical Institutions (CFTIs), etc. in the next academic year (2020-2021).The First JEE Main-2020 will be conducted in January 2020 between 6th January (Monday) and 11th January (Saturday) 2020 and the Second JEE Main-2020 will be conducted between 3rd April (Friday) and 9th April (Thursday) 2020.

As per the eligibility criteria for B.Arch and B.Planning courses and according to the opinion of Subject Experts, a few changes in the pattern of the question paper(s) and number of question(s) for B.E./B.Tech, B.Arch and B.Planning have been approved by the JEE Apex Board (JAB) for the conduct of JEE Main-2020 Examination.

The pattern of question paper(s) is given below:

PAPER	SUBJECTS	NO. OF QUESTIONS	TYPE OF QUESTIONS	TIMING OF THE EXAMINATION(IST)	
				First Shift	Second Shift
B.E./B.Tech.	Mathematics	25(20+5)	20 Objective Type - Multiple Choice Questions (MCQs) & 5 Questions with answer as numerical value, with equal weightage to Mathematics, Physics & Chemistry	09:30 a.m. to 12:30 p.m.	02:30 p.m. to 05:30 p.m.
	Physics	25(20+5)			
	Chemistry	25(20+5)			
B. Arch.	Mathematics – Part I	25(20+5)	20 Objective Type - Multiple Choice Questions (MCQs) & 5 Questions for which answer is a numerical value	09:30 a.m. to 12:30 p.m.	02:30 p.m. to 05:30 p.m.
	Aptitude Test – Part II	50	Objective Type - Multiple Choice Questions (MCQs)		
	Drawing Test – Part III	2	Questions for drawing test		
B. Planning	Mathematics – Part I	25(20+5)	20 Objective Type - Multiple Choice Questions (MCQs) & 5 Questions for which answer is a numerical value	02:30 p.m. to 05:30 p.m.	
	Aptitude Test – Part II	50	Objective Type - Multiple Choice Questions (MCQs)		
	Planning Based Questions - Part III	25	Objective Type - Multiple Choice Questions (MCQs)		

Note: The timing for those Candidates who opt for B.Arch. and B.Planning (both) will be 02:30 p.m. to 06:00 p.m. (IST)

The above Examinations will be held in "Computer Based Test" (CBT) Mode only, except that the Drawing Test for B.Arch. will be held in "Pen & Paper" (offline) mode. A candidate may appear in B.E./B.Tech, B.Arch and B.Planning depending upon the course/s he/she wishes to pursue.

The candidates aspiring to take admission to the undergraduate programs at IITs for the year 2020 will also have to appear in B. E. /B. Tech. Paper of JEE Main -2020. Based on the performance in the B.E./B. Tech. of JEE Main-2020, number of top candidates as per the requirement for JEE Advanced-2020 will be eligible to appear in JEE Advanced-2020. Admission to IITs will be based on category -wise All India Rank (AIR) in JEE Advanced,subject to the conditions as would be mentioned in JEE Advanced-2020 website.

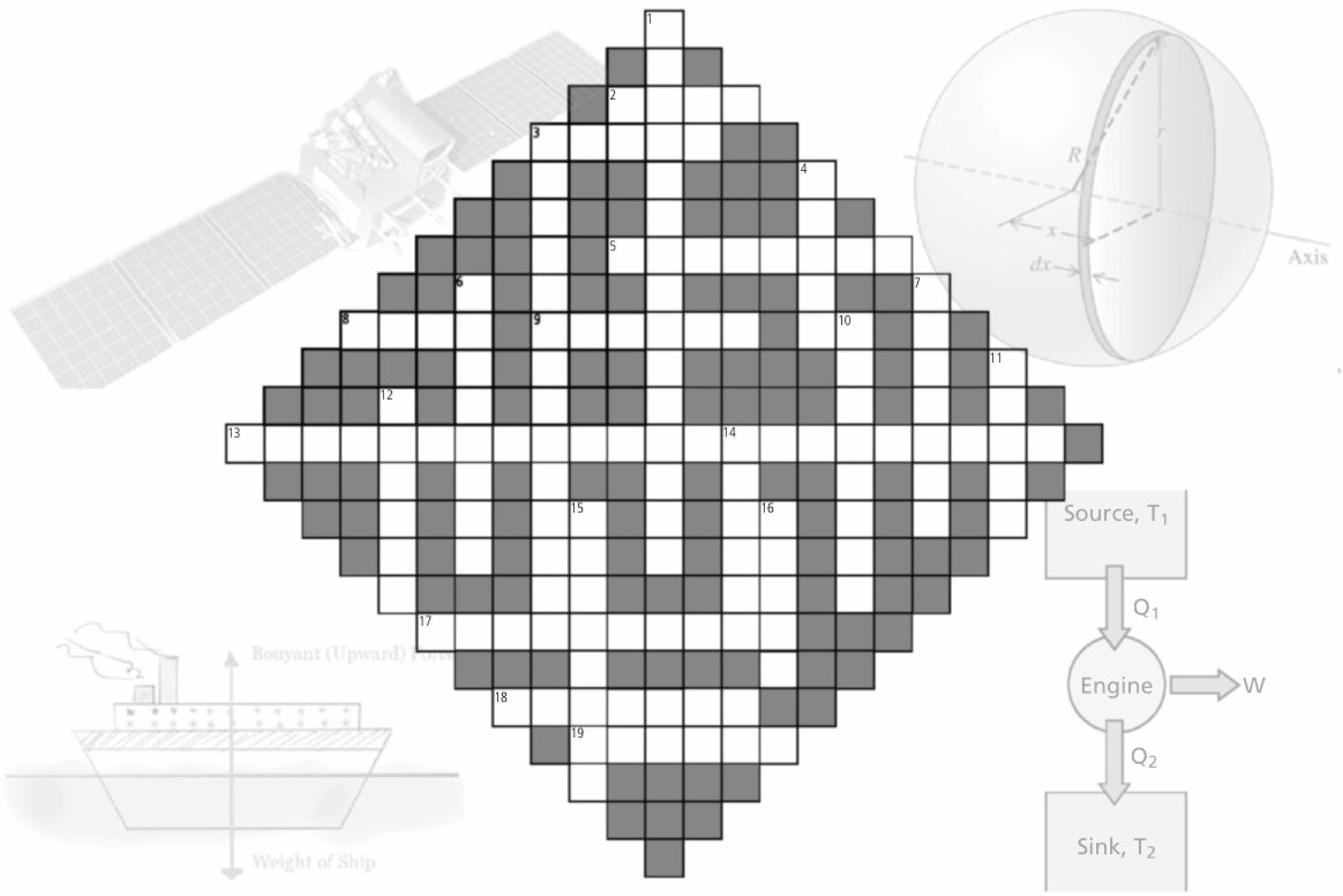
For the April JEE Main-2020, a separate notice will be issued later on and the candidates will be required to apply separately. However, candidates are not required to compulsorily appear in both the tests i.e. January JEE Main-2020 and April JEE Main-2020. In case, a candidate appears in both the tests, the better of the two scores will be used for the admissions and eligibility of JEE Advanced-2020.

For more information visit www.jeemain.nic.in

CROSS WORD

CLASS XI

Readers can send solutions at editor@mtg.in or post us with their complete address by 10th of every month to get their names published in next issue.



DOWN

1. Material which do not break well beyond the elastic limit **(7,8)**
3. Angle enclosed between the tangents to the liquid surface and the solid surface inside the liquid. **(5, 2, 7)**
4. The sound which has non pleasing effect on ears **(5)**
6. In an isochoric process, volume remains **(8)**
7. The product of force and time **(7)**
10. Motion only under the force of gravity **(4, 4)**
11. Characteristic feature which differentiate a sharp and a flat sound **(5)**
12. Restoring force per unit area is called **(6)**
14. Ratio of change in configuration to the original configuration **(6)**
15. Quantity which is conserved during collision **(8)**
16. The rate at which the body can do the work **(5)**

ACROSS

2. Reflection of sound waves **(4)**
3. The liquid rises in a capillary tube when angle of contact is **(5)**
5. The temperature below which water vapour begins to condense and dews are formed **(3, 5)**
8. Work done by centripetal force **(4)**
9. Tone whose frequency is double the fundamental frequency **(6)**
13. The satellite which appears stationary relative to earth **(13, 9)**
17. It converts heat into work through a cyclic process **(4, 6)**
18. A measure of disorder of molecular motion of a system **(7)**
19. When air blows over a roof the force on the roof is **(6)**

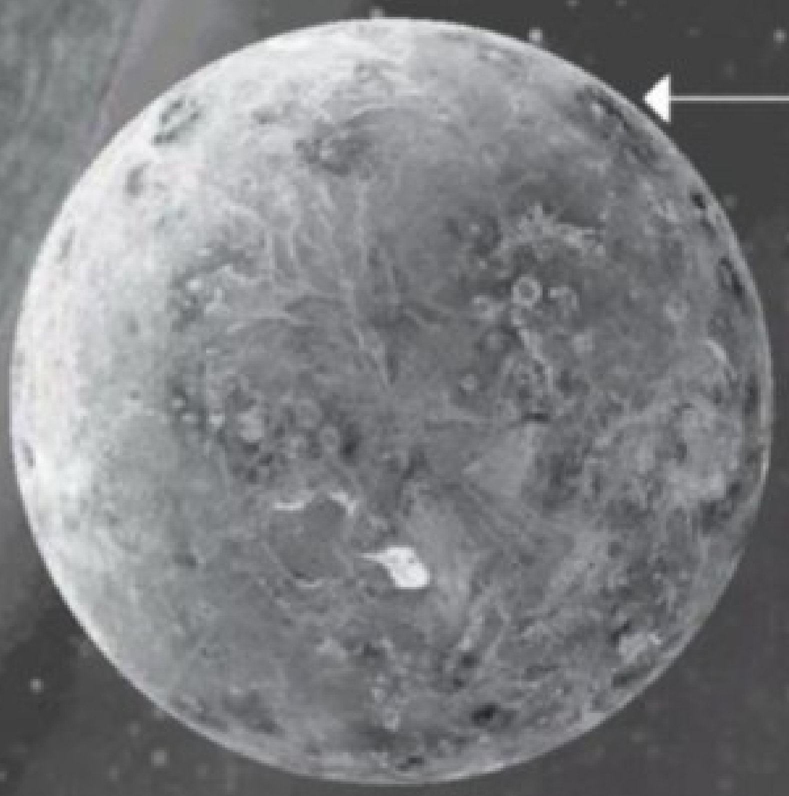
Giant Leaps Ahead

The Indian Space Research Organisation (ISRO) lost contact with its lunar lander and rover but experts say the Chandrayaan-2 is "95% successful" as the mission's space probe has been put in its orbit around the moon. It can send back valuable data that will help ISRO's future missions. Here is a look at the missions the space agency has lined up in the coming years

Reaching for the Sun

Aditya-1, 2019-2020

Aditya-I is India's first dedicated scientific mission to study the Sun. A 400 kg class space telescope will be inserted into a halo orbit 1.5 million km from the Earth to study the three layers of the sun — photosphere, chromosphere and corona, the outer atmosphere of the star in our solar system. The mission is aimed at developing insights on the weather in space and to understand why the outer atmosphere of the Sun is 200 times hotter than the solar disc. The satellite is expected to be launched by next year.



Befriending a solar sibling

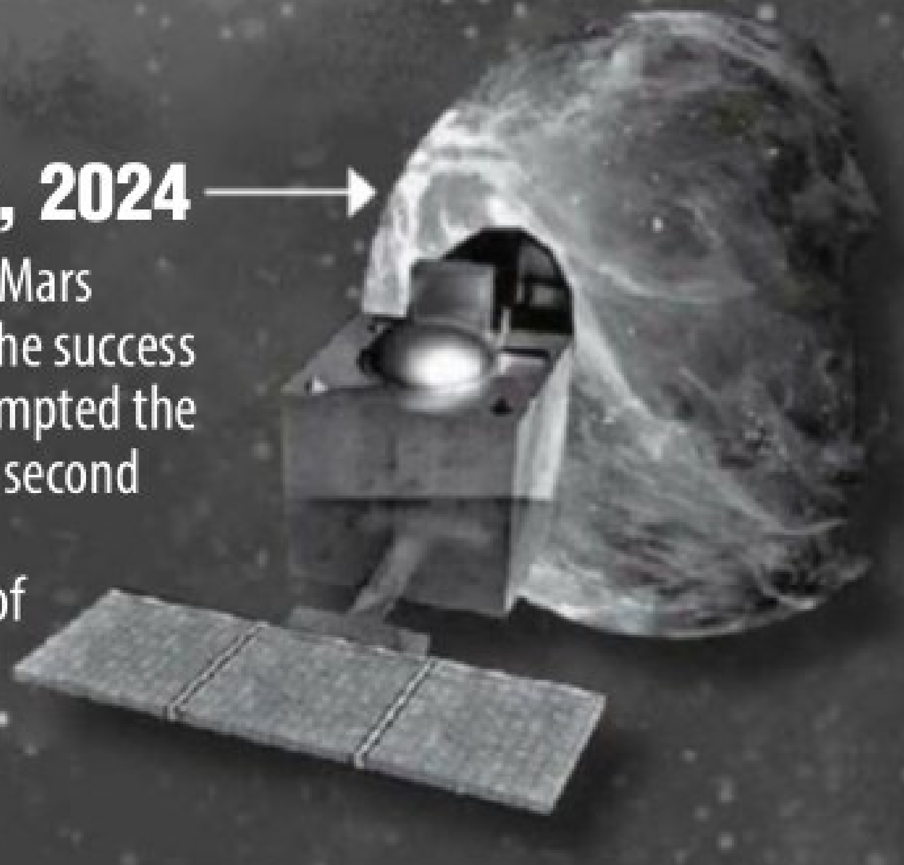
Mission Venus, 2023

ISRO is planning a mission to the Earth's "twin sister", Venus. Both the Earth and Venus share similarities in size, mass, density, bulk composition and gravity. The space agency will fly a spacecraft around 400 km over Venus to conduct research and understand its formation, its atmosphere and its interaction with the solar wind. The mission is expected by 2023.

Back to Mars

Mangalyaan-2, 2024

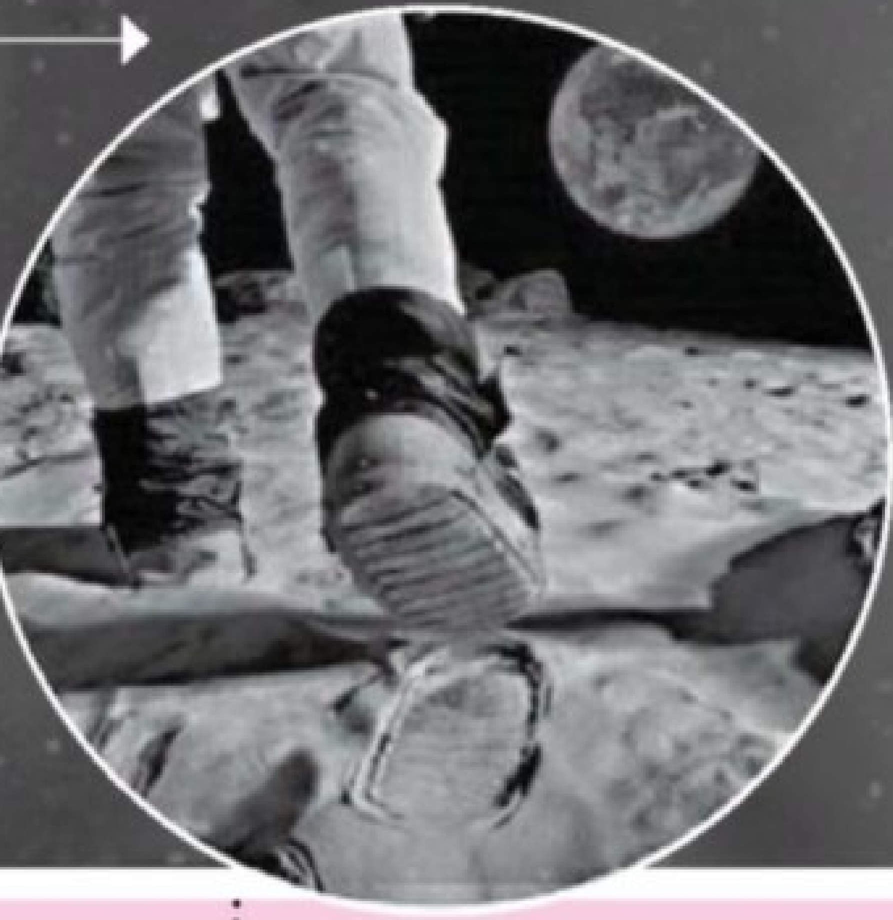
ISRO plans to return to Mars through this mission. The success of Mangalyaan has prompted the space agency to send a second probe by 2024, which will do deeper studies of the Earth's neighbour and understand the evolution of the red planet better.



Drilling with Japan

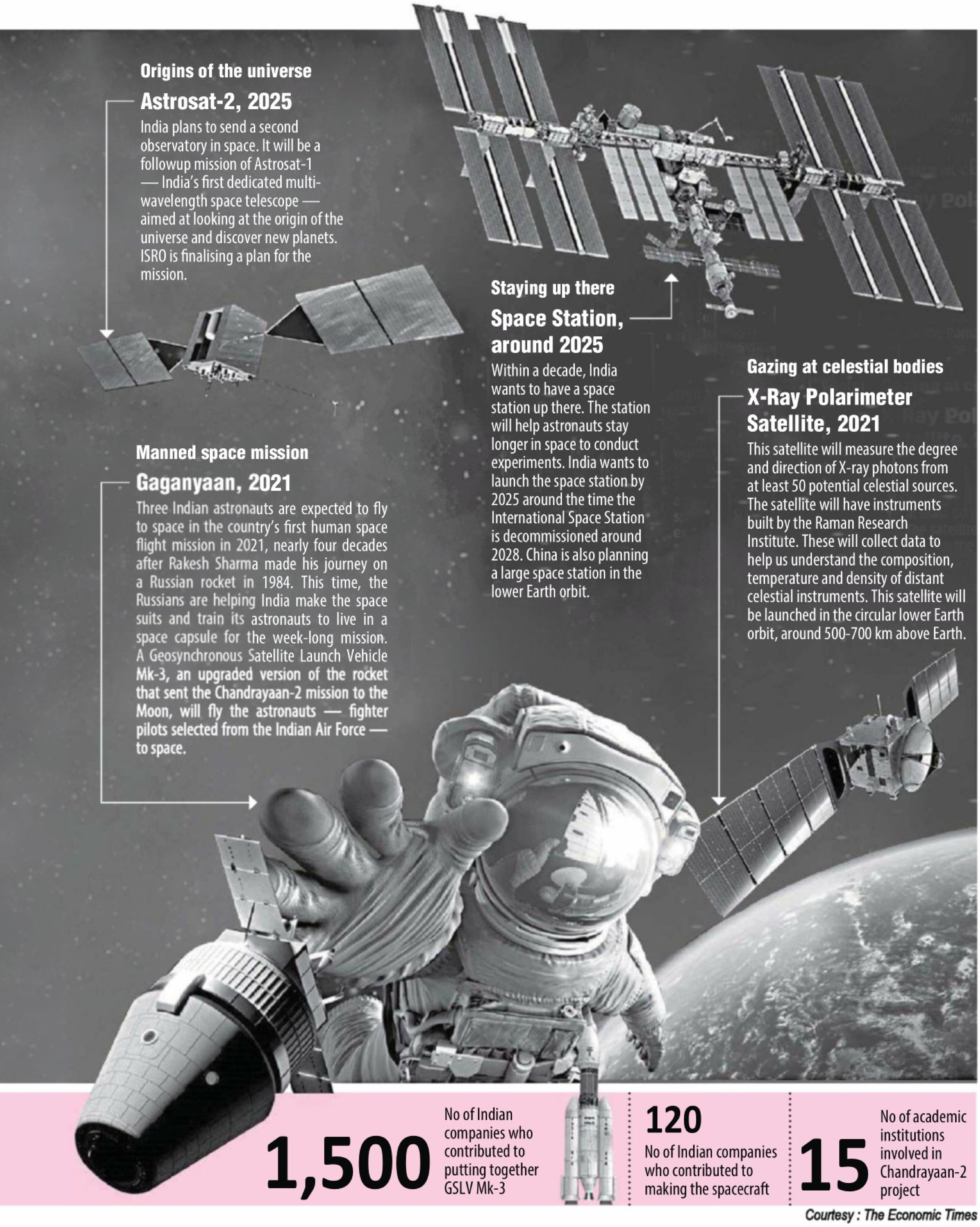
Moon Mission, 2023

ISRO and Japan Aerospace Exploration Agency will send a joint mission to the Moon's south pole. The mission includes landing a rover that will drill the Moon's surface to conduct scientific experiments. The primary focus will be to explore the existence of water. Japan is likely to provide the rocket and a lunar rover, while India is likely to contribute with a lander for the mission, a follow-up of Chandrayaan-2. It would explore the suitability of the region for establishing a sustainable lunar base.



Facts about Chandrayaan-2

₹603 cr	₹375 cr	3.8 tonnes
Mission cost of Chandrayaan-2, India's second mission to the Moon	Cost of the launcher, GSLV Mk-3, India's heaviest rocket	Weight of integrated module with orbiter, lander and rover, including 14 instruments



Origins of the universe

Astrosat-2, 2025

India plans to send a second observatory in space. It will be a followup mission of Astrosat-1 — India's first dedicated multi-wavelength space telescope — aimed at looking at the origin of the universe and discover new planets. ISRO is finalising a plan for the mission.

Staying up there

Space Station, around 2025

Within a decade, India wants to have a space station up there. The station will help astronauts stay longer in space to conduct experiments. India wants to launch the space station by 2025 around the time the International Space Station is decommissioned around 2028. China is also planning a large space station in the lower Earth orbit.

Gazing at celestial bodies

X-Ray Polarimeter Satellite, 2021

This satellite will measure the degree and direction of X-ray photons from at least 50 potential celestial sources. The satellite will have instruments built by the Raman Research Institute. These will collect data to help us understand the composition, temperature and density of distant celestial instruments. This satellite will be launched in the circular lower Earth orbit, around 500-700 km above Earth.

Manned space mission

Gaganyaan, 2021

Three Indian astronauts are expected to fly to space in the country's first human space flight mission in 2021, nearly four decades after Rakesh Sharma made his journey on a Russian rocket in 1984. This time, the Russians are helping India make the space suits and train its astronauts to live in a space capsule for the week-long mission. A Geosynchronous Satellite Launch Vehicle Mk-3, an upgraded version of the rocket that sent the Chandrayaan-2 mission to the Moon, will fly the astronauts — fighter pilots selected from the Indian Air Force — to space.

1,500 No of Indian companies who contributed to putting together GSLV Mk-3

120 No of Indian companies who contributed to making the spacecraft

15 No of academic institutions involved in Chandrayaan-2 project

Courtesy : The Economic Times

PHYSICS MUSING

SOLUTION SET-74

1. **(c) :** Let required minimum velocity of m be v_o.
Velocity of second bead after collision will be

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1}{m_1 + m_2}u_1 \qquad (\because\ u_2 = 0)$$

$$= \frac{2m}{m + \frac{m}{2}}v_0 \quad \text{or} \quad v_2 = \frac{4}{3}v_o$$

Similarly, you get of third bead after collision as

$$v_3 = \left(\frac{4}{3}\right)^2 v_o$$

So, velocity of n^{th} bead after collision will be

$$v_n = \left(\frac{4}{3}\right)^{n-1} v_o$$

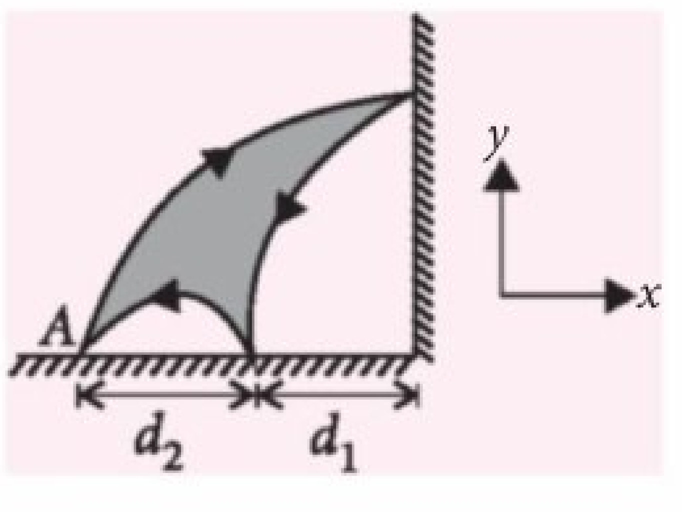
But $(v_n)_{\min} = \sqrt{4gR} \Rightarrow \sqrt{4gR} = \left(\frac{4}{3}\right)^{n-1} v_o$

$$\Rightarrow v_o = \left(\frac{3}{4}\right)^{n-1}\sqrt{4gR} = \left(\frac{3}{4}\right)^{n-1} 2\sqrt{gR}$$

2. **(a) :** $R = \dfrac{2u_x u_y}{g}$ and $T = \dfrac{2u_y}{g}$

After 1[st] collision, $v_x = eu_x$

After 1[st] collision,



$$d_1 = v_x\frac{T}{2} = eu_x\frac{T}{2}$$

$$= \frac{eu_x}{2}\cdot\frac{2u_y}{g} = \frac{eu_x u_y}{g}$$

After 2[nd] collision, $v'_x = v_x = eu_x$

And $v'_y = eu_y$

$$\therefore \quad d_2 = \frac{2v'_x v'_y}{g} = \frac{2(eu_x)(eu_y)}{g} = \frac{2e^2 u_x u_y}{g}$$

Now, $d_1 + d_2 = \dfrac{eu_x u_y}{g}(1 + 2e)$

But $d_1 + d_2 = \dfrac{R}{2} \quad \therefore \quad \dfrac{eu_x u_y}{g}(1+2e) = \dfrac{2u_x u_y}{2g}$

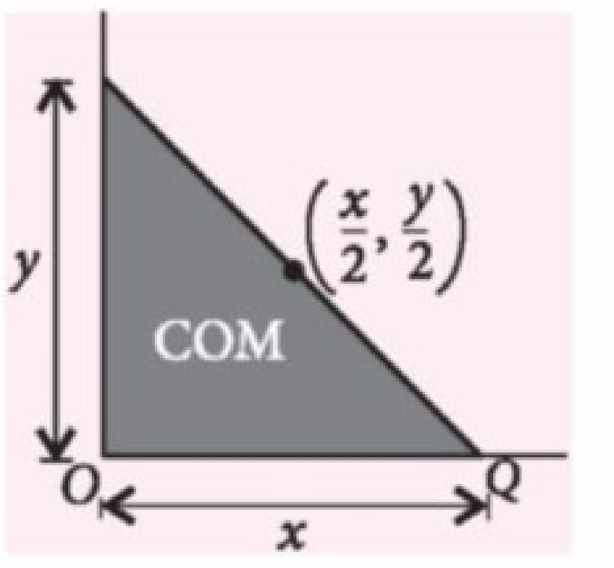
$e + 2e^2 = 1 \Rightarrow 2e^2 + e - 1 = 0$

By solving, we get, $e = \dfrac{1}{2} = 0.5$

3. **(b) :** $y = \sqrt{L^2 - x^2} \Rightarrow \dfrac{dy}{dt} = -\dfrac{x}{\sqrt{L^2 - x^2}}\cdot\dfrac{dx}{dt}$

or $\quad v_y = -\dfrac{3}{\sqrt{5^2 - 3^2}}(2)$

$$= -\frac{3}{2}\ \text{m s}^{-1}$$



At any moment coordinates of centre of mass are $\left(\dfrac{x}{2}, \dfrac{y}{2}\right)$

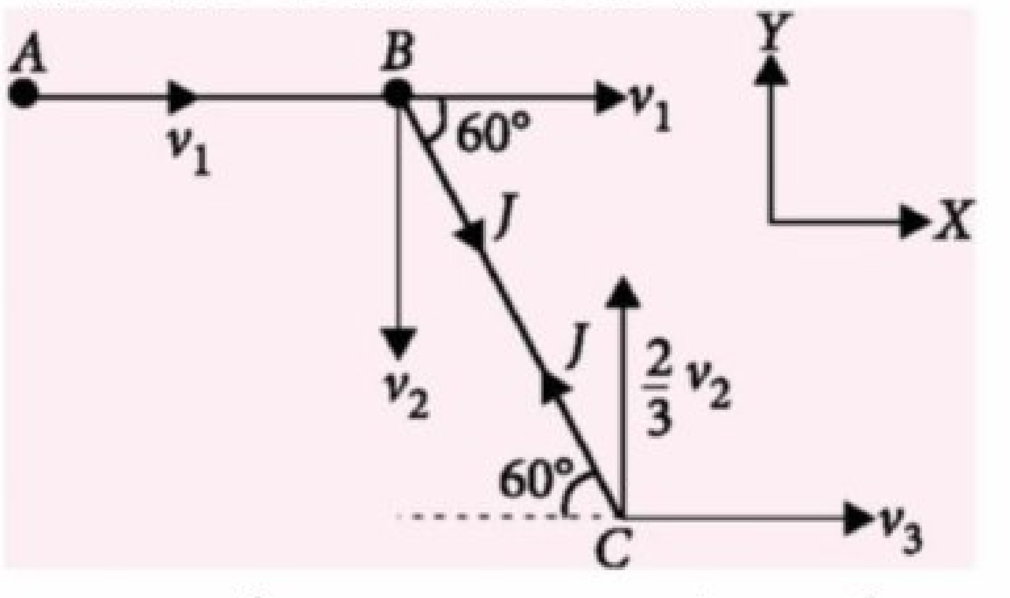
Now, $(v_{\text{COM}})_x = \dfrac{d}{dt}\left(\dfrac{x}{2}\right) = \dfrac{1}{2}\cdot\dfrac{dx}{dt} = \dfrac{1}{2}(2) = 1\ \text{m s}^{-1}$

$$(v_{\text{COM}})_y = \frac{d}{dt}\left(\frac{y}{2}\right) = \frac{1}{2}\cdot\frac{dy}{dt} = \frac{1}{2}\left(-\frac{3}{2}\right) = -\frac{3}{4}\text{m s}^{-1}$$

(Negative sign shows that velocity of rod is along negative y axis.)

$$v_{\text{COM}} = \sqrt{(v_{\text{COM}})_x^2 + (v_{\text{COM}})_y^2} = \sqrt{1 + \frac{9}{16}} = \frac{5}{4} = 1.25\ \text{m s}^{-1}$$

4. **(d) :** As C moves, the string BC is slack and it gets taut and impulse acts along BC, when BC makes an angle 60° with AB as shown below.



For BC system, from conservation of moment in Y-direction, we have

$$0 = 3m(v_C)_y + 2m(v_B)_y \Rightarrow (v_C)_y = -\frac{2}{3}(v_B)_y$$

If v_1 and $-v_2$ be x and y-components of final velocity of B then

$$(v_B)_y = -v_2 \Rightarrow (v_C)_y = \frac{2}{3}v_2$$

For $(A + B + C)$ systems, from conservation of momentum along X-direction, we have

$3mv_0 = mv_1 + 2mv_1 + 3mv_3$

$\Rightarrow v_0 = v_1 + v_3$ ...(i)

For particles B and C system, from components of velocities along BC will be equal,

$$v_1\cos 60° + v_2\cos 30° = -\frac{2}{3}v_2\cos 30° + v_3\cos 60°$$

$$\Rightarrow \frac{v_1}{2} + \frac{\sqrt{3}}{2}v_2 = \frac{v_3}{2} - \frac{v_2}{\sqrt{3}} \text{ or } v_1 - v_3 + \frac{5}{\sqrt{3}}v_2 = 0 \quad \text{...(ii)}$$

Since, impulse, J = change in momentum, we can write,

For C along X-axis,

$$J\cos 60° = 3m(v_0 - v_3) \Rightarrow \frac{J}{2} = 3m(v_0 - v_3) \quad \text{...(iii)}$$

for C along Y-axis,

$$J\cos 30° = 3m\left(\frac{2}{3}\cdot v_2\right) = 2mv_2 \Rightarrow \frac{\sqrt{3}}{2}J = 2mv_2 \quad \text{...(iv)}$$

On solving equations (i), (ii), (iii) and (iv), we get

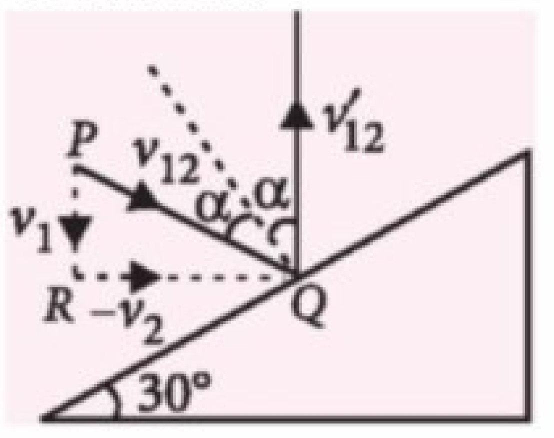
$$v_1 = \frac{2}{19} v_0$$

5. (0.58) : In the figure, v_{12} = velocity of ball with respect to wedge before collision and v'_{12} = velocity of ball with respect to wedge after collision.

In elastic collision v_{12} and v'_{12} will make equal angle (say α) with the normal to the plane.

We can show that $\alpha = 30°$

$\therefore \angle PQR = 30°$



$$\Rightarrow \quad \tan 30° = \frac{v_1}{v_2} \quad \Rightarrow \quad \frac{v_1}{v_2} = \frac{1}{\sqrt{3}} \simeq 0.58$$

6. (3.00) : Along perpendicular to the plane,

$$0 = u\sin(\phi - 45°)t - \frac{1}{2}(g\cos 45°)t^2$$

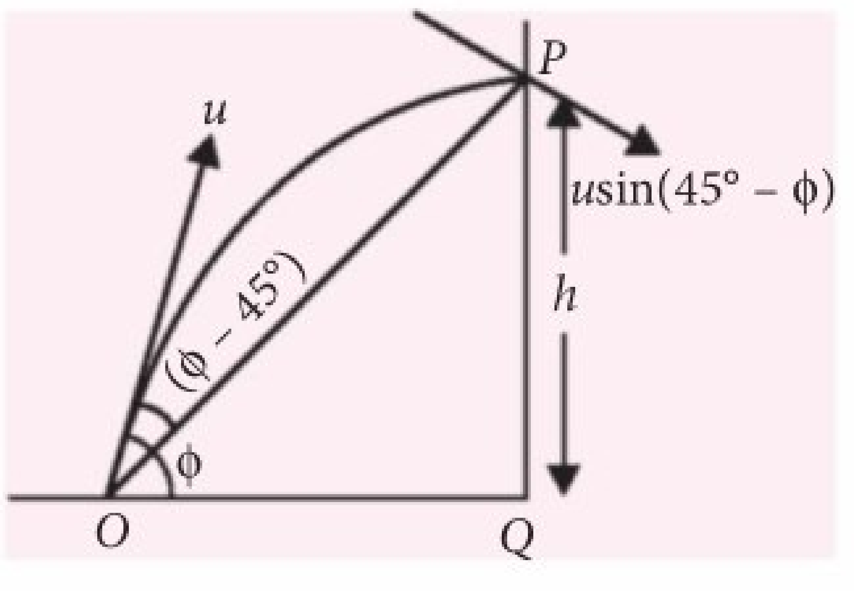
$$\text{or } t = \frac{2\sqrt{2}\,u}{g}\sin(\phi - 45°).$$

Along the plane, $u\cos(\phi - 45°) = (g\sin 45°)t$

$$u\cos(\phi - 45°) = \frac{g}{\sqrt{2}}\left[\frac{2\sqrt{2}\,u}{g}\sin(\phi - 45°)\right]$$

or $u\cos(\phi - 45°)$

$= 2u\sin(\phi - 45°)$



$$\text{or } \frac{1}{2} = \tan(\phi - 45°)$$

$$= \frac{\tan\phi - 1}{1 + \tan\phi}$$

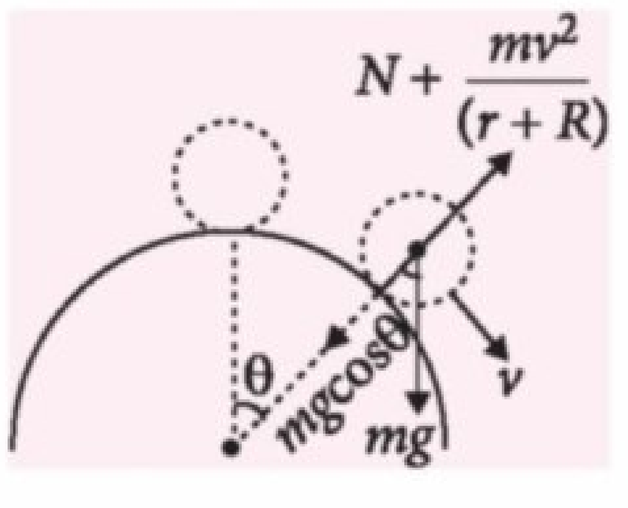
Solving, we get

$\tan\phi = 3$

7. (17.00) : If at angle θ, ball breaks off, then

$$mg\cos\theta = 0 + \frac{mv^2}{r+R}$$

[Here $N = 0$ at the point of break off]



$$g\cos\theta = \frac{v^2}{r+R}$$

By work-energy theorem,

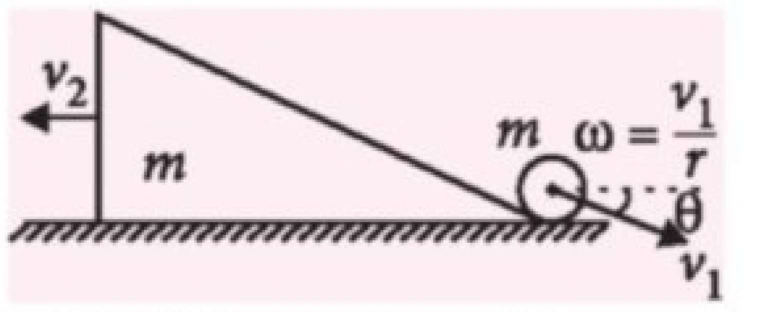
$$0 + mg(R+r)(1-\cos\theta) = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2$$

$$g(R+r)(1-\cos\theta) = \frac{7}{10}v^2$$

$$g(R+r) - v^2 = \frac{7}{10}v^2 \quad \Rightarrow \quad v = \sqrt{\frac{10g(R+r)}{17}}$$

$$\omega = \frac{v}{r} = \sqrt{\frac{10g(R+r)}{17 \times r^2}}$$

8. As no external force is acting in horizontal direction on the system, linear momentum of the system can be conserved along x direction.



$$mv_2 = m(v_1\cos\theta - v_2) \quad \Rightarrow \quad v_1 = \frac{2v_2}{\cos\theta}$$

By work-energy theorem,

$$mgh = \frac{1}{2}mv_2^2 + \frac{1}{2}m[(v_1\cos\theta - v_2)^2 + (v_1\sin\theta)^2] + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$2gh = v_2^2 + v_1^2 + v_2^2 - 2v_1v_2\cos\theta + \frac{2}{5}v_1^2$$

v_1 = velocity of ball with respect to wedge

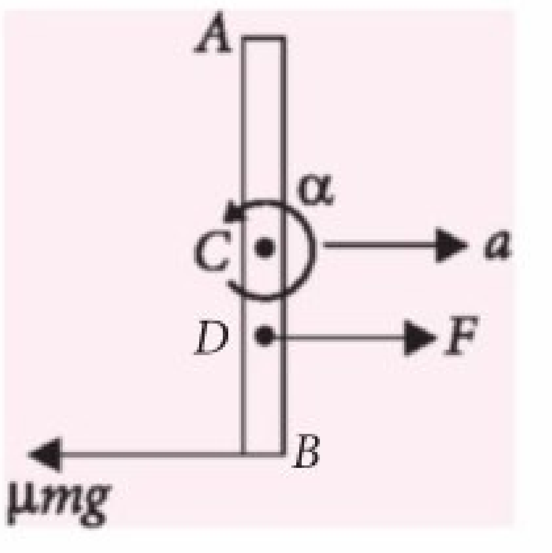
$$2gh = 2v_2^2 + \frac{7}{5}\left(\frac{4v_2^2}{\cos^2\theta}\right) - 4v_2^2 \quad \Rightarrow v_2 = \sqrt{\frac{5gh}{14\sec^2\theta - 5}}$$

9. If a is the acceleration of rod, then net force acting on rod is,

$F - \mu mg = ma$

$$\Rightarrow \quad a = \frac{F}{m} - \mu g$$

If α is the angular acceleration of the rod, we use



$$F\left(\frac{l}{3}\right) - \mu mg\left(\frac{l}{2}\right) = \frac{ml^2}{12}.\alpha \quad \Rightarrow \quad \alpha = \frac{4F}{ml} - \frac{6\mu g}{l}$$

Acceleration of point A on rod

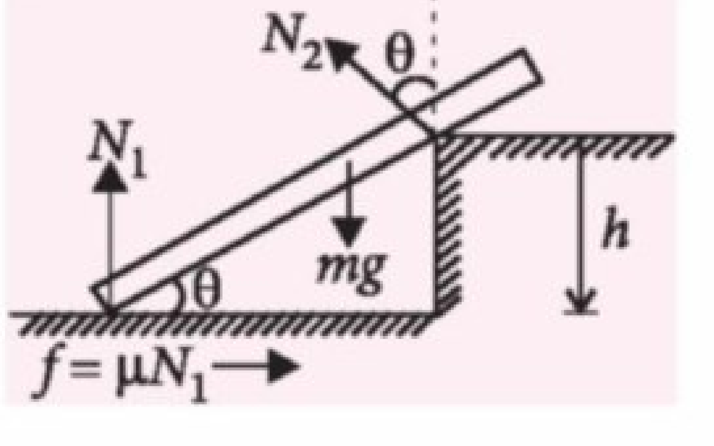
$$a_A = a - \frac{l\alpha}{2} = \left(\frac{F}{m} - \mu g\right) - \frac{l}{2}\left(\frac{4F}{ml} - \frac{6\mu g}{l}\right)$$

$$a_A = 2\mu g - \frac{F}{m}$$

10. For translational equilibrium of rod, we have, $N_2\sin\theta = \mu N_1$ ...(i)

$N_1 + N_2\cos\theta = mg$ ...(ii)

For rotational equilibrium of rod we get,



$$\tau_A = mg \times \frac{L}{2}\cos\theta - N_2 \times \frac{h}{\sin\theta} = 0$$

$$N_2 = \frac{mgL\sin\theta\cos\theta}{2h}$$

$$\text{From eqn. (ii), } N_1 = mg - \frac{mgL}{2h}\sin\theta\cos^2\theta$$

$$\text{From eqn. (i), } \mu = \frac{N_2\sin\theta}{N_1}$$

$$= \frac{\frac{mgL}{2h}\sin^2\theta\cos\theta}{mg - \frac{mgL}{2h}\sin\theta\cos^2\theta} = \frac{L\sin^2\theta\cos\theta}{2h - L\sin\theta\cos^2\theta}$$

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